



# GPU-accelerated data expansion for the Marching Cubes algorithm

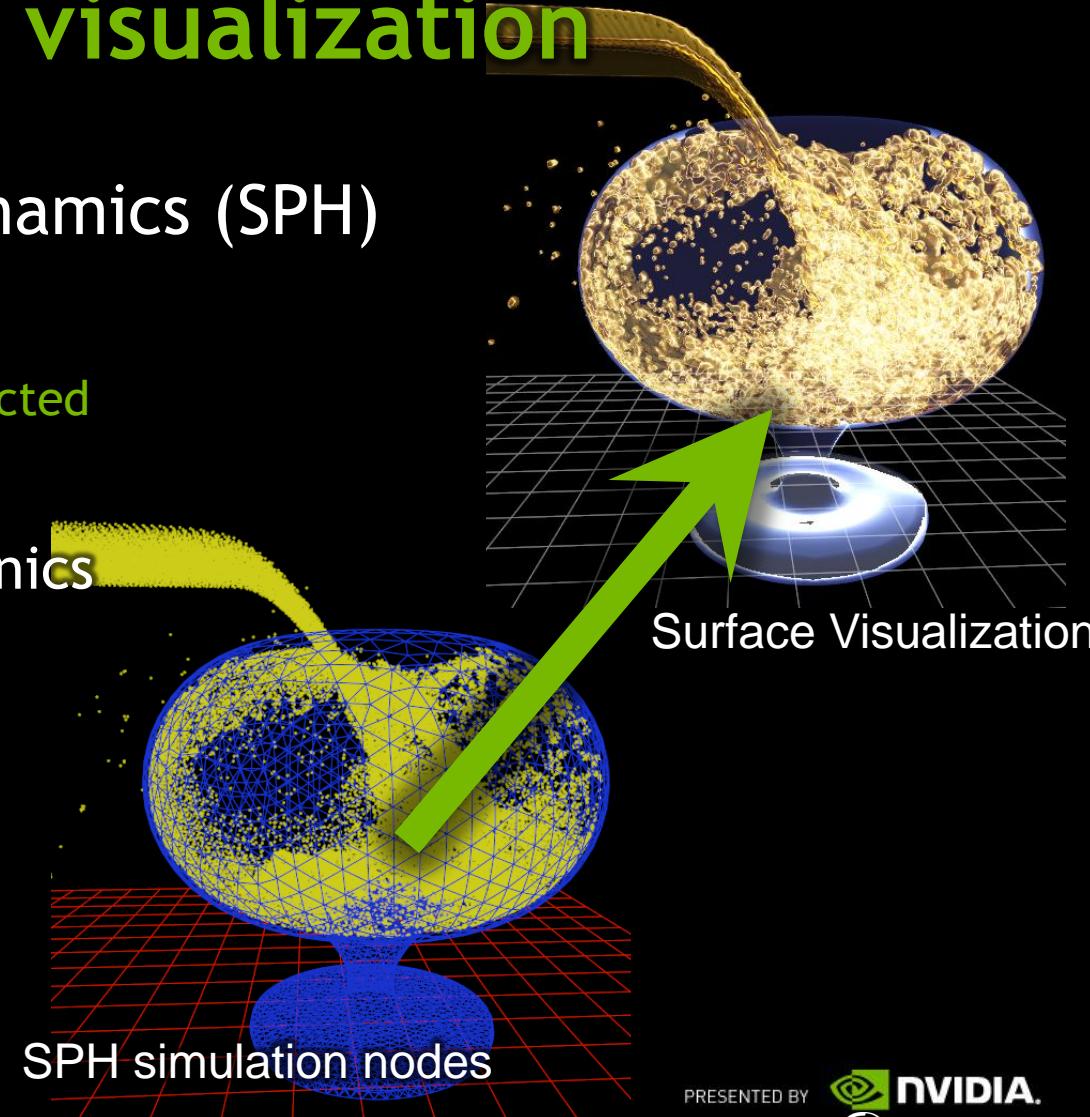
San Jose (CA) | September 23rd, 2010  
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# Agenda

- Motivation & Background
- Data Compaction and Expansion
  - Histogram Pyramid algorithm and its variations
  - Optimizations and benchmark results
- Marching Cubes based on Histogram Pyramids
  - Mapping and performance considerations
  - Benchmark results
- Visualization of SPH simulation results
  - Videos

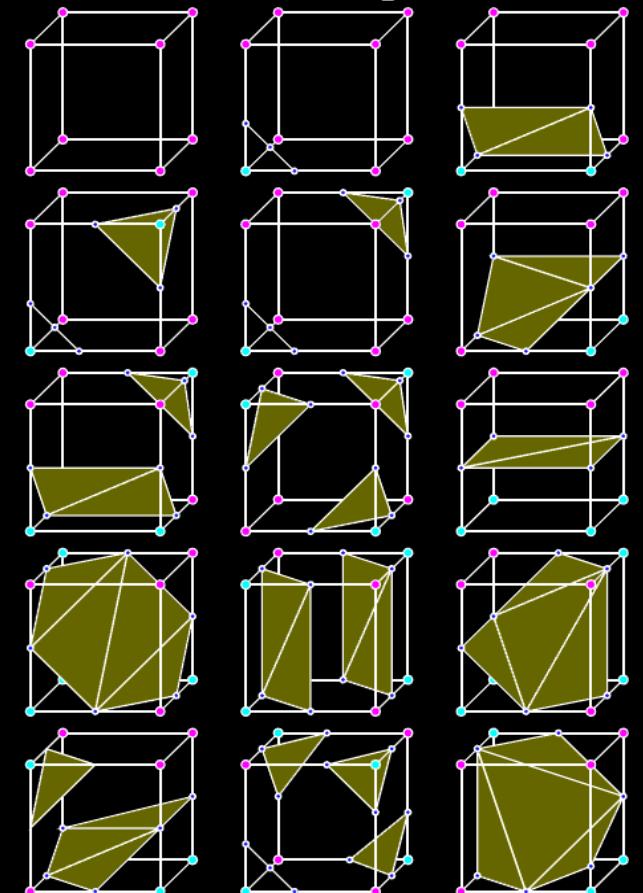
# Motivation: Fast SPH visualization

- Smoothed-particle Hydrodynamics (SPH)
  - Meshless Lagrangian method:
    - Nodes (particles) are **not connected**
    - Node position **varies with time**
  - Models **fluid** and **solid** mechanics
  - Nodes form a **density field**
- High-quality visualization:
  1. Approximate density field
  2. **Marching Cubes**
  3. Render iso-surface

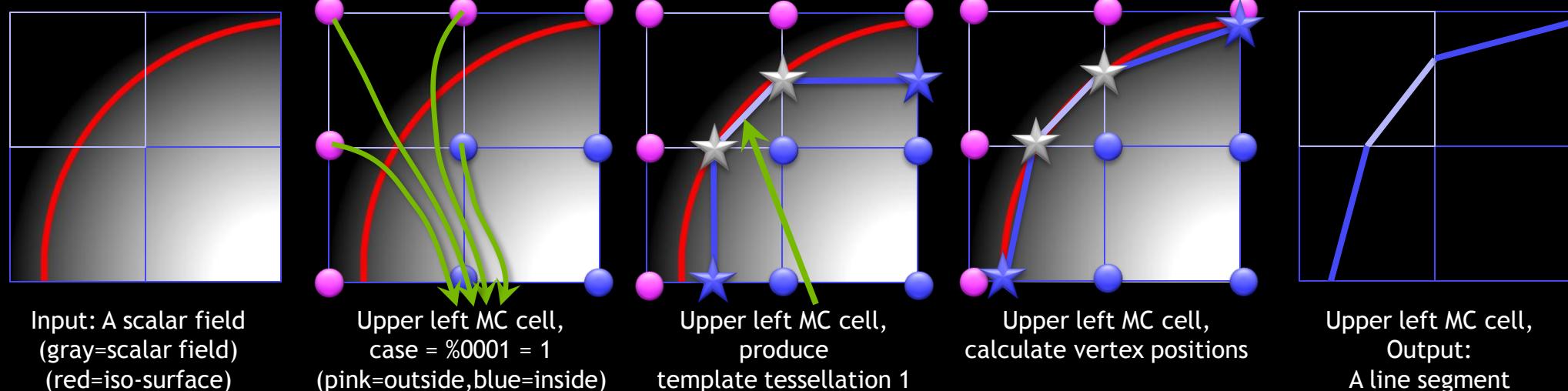


# Extract iso-surface via Marching Cubes

- Scalar field is sampled over 3D grid
- Marching Cubes [Lorensen87]
  - Marches through a **regular 3D grid of cells**
  - 1. Each MC cell spans 8 samples
  - 2. Label corners as **inside or outside iso-value**
  - 3. Eight in/out labels give **256** possible cases
  - 4. Each case has a **tessellation template**
    - Devised such that **tessellations of adjacent cells match**
    - Vertices lie on lattice edges
      - positioned using linear interpolation
  - De-facto standard algorithm for this problem



# Example: Marching Cubes in 2D



1. For each cell:  
Determine MC case and # vertices of template
2. Determine total # vertices and output index of each MC cell's vertices
3. During vertex output: calculate actual positions

✓ Data-parallel!

Not trivially data-parallel!

✓ Data-parallel!

# Step 2 is Data Compaction & Expansion

- We want to answer:
  - How many triangles to draw?
  - What is the **mapping** between input and output?
    - **Classic:** At which output position  $j$  shall MC cell  $i$  write vertex  $k$ ?
    - **Put differently:** Which MC cell  $i$  and vertex  $k$  does output position  $j$  belong to?
- Data compaction & expansion provide answers:
  - **Data compaction:**
    - Extract all cells that produce geometry
  - **Data expansion:**
    - Each cell that produces geometry issues 3-15 vertices

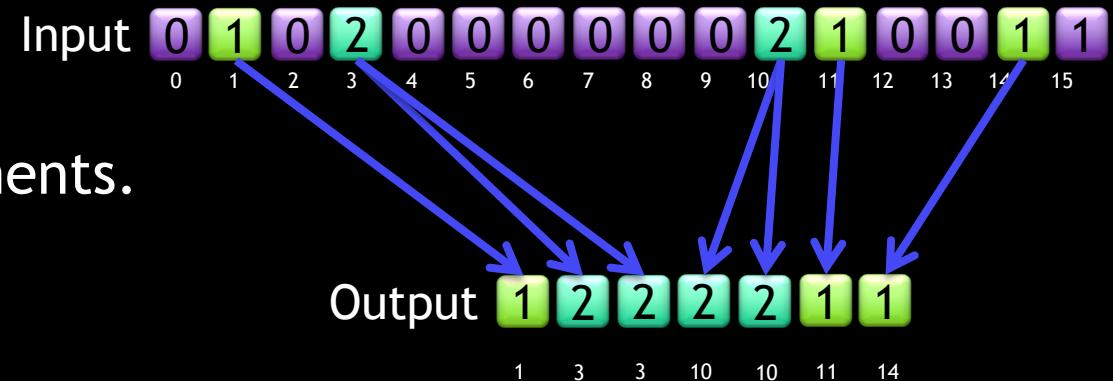
# Data Compaction and Expansion

- Problem definition

- We start with  $n$  input elements.
- Input element  $j$  produces  $a_j$  output elements.
- Discard all elements where  $a_j = 0$ .

- An important algorithmic pattern!

- Trivial implementation in serial implementation (e.g. CPU).
- **Non-trivial** on data-parallel architectures (e.g. GPU)!



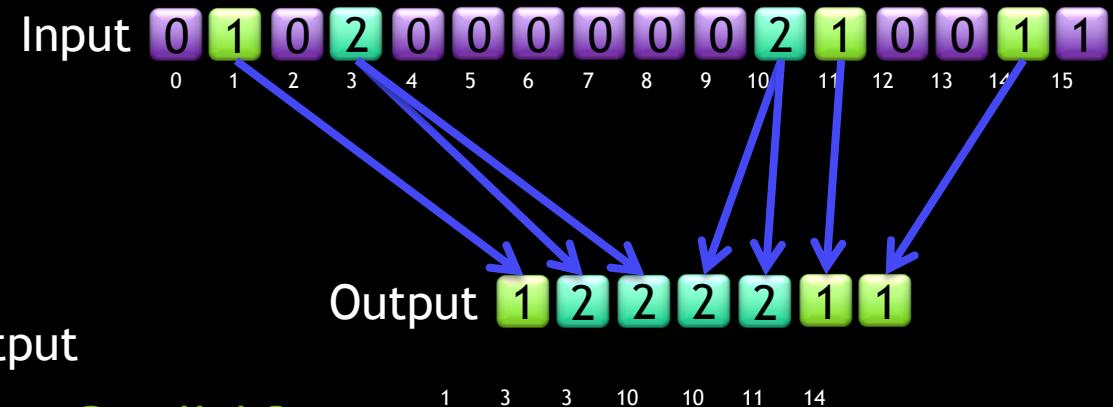
# Input or Output-centric solutions

- Input-centric solution:

- For every input element
    - Compute output offsets
    - *Scatter* relevant input to output
    - Typical serial solution and *Data-Parallel Scan*

- Output-centric solution:

- For every output element
    - *Determine* input element from output index
    - Histogram Pyramid (*HistoPyramid*): Reduction-based search structure



# HistoPyramid: Stages of Algorithm

- *Input is Baselevel*

- For each input element, init with number of output elements



- *Level Buildup*

- Build further levels through reduction

- *HistoPyramid Traversal*

- For each output index:  
Find corresponding input index (via HistoPyramid traversal)

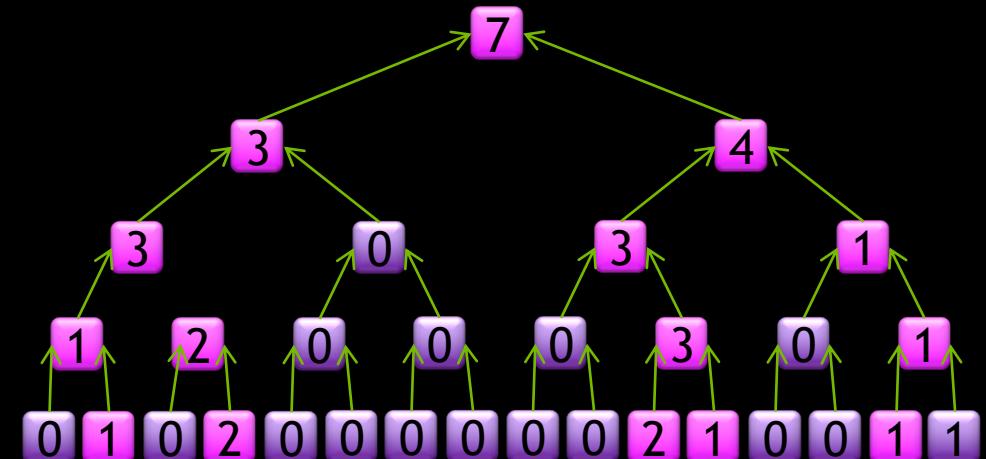
# HistoPyramid Buildup

- Build further levels from baselevel
  - Add two elements (reduction)

- Number of elements halves each iteration
- $\log_2 n$  iterations
  - Each iteration half the size of the previous iteration

- Data-Parallel algorithm

- Top element equals number of output elements (Step 2A)
- Data of all reduction levels: 2:1 HistoPyramid

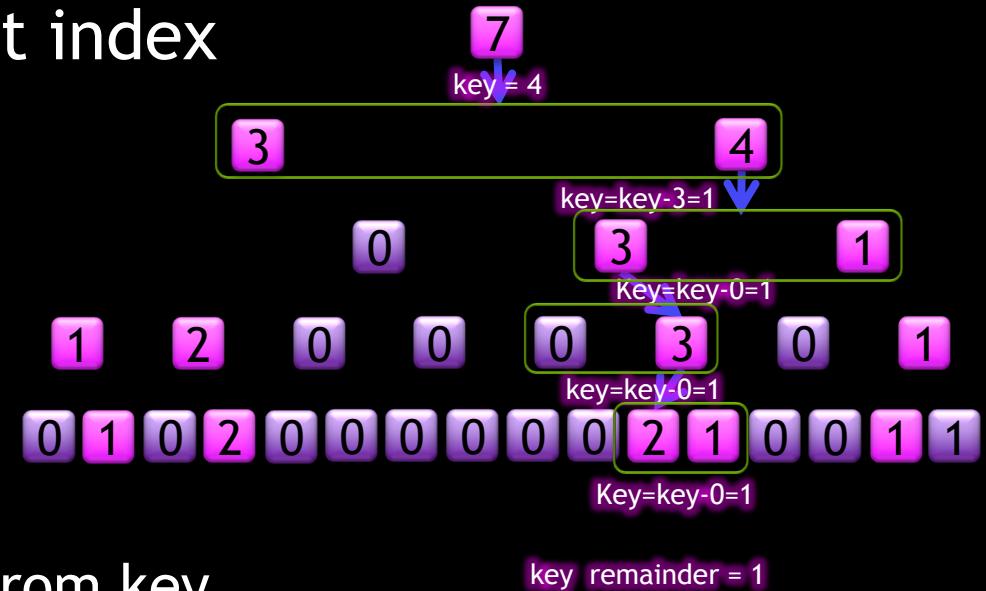


# Output Allocation

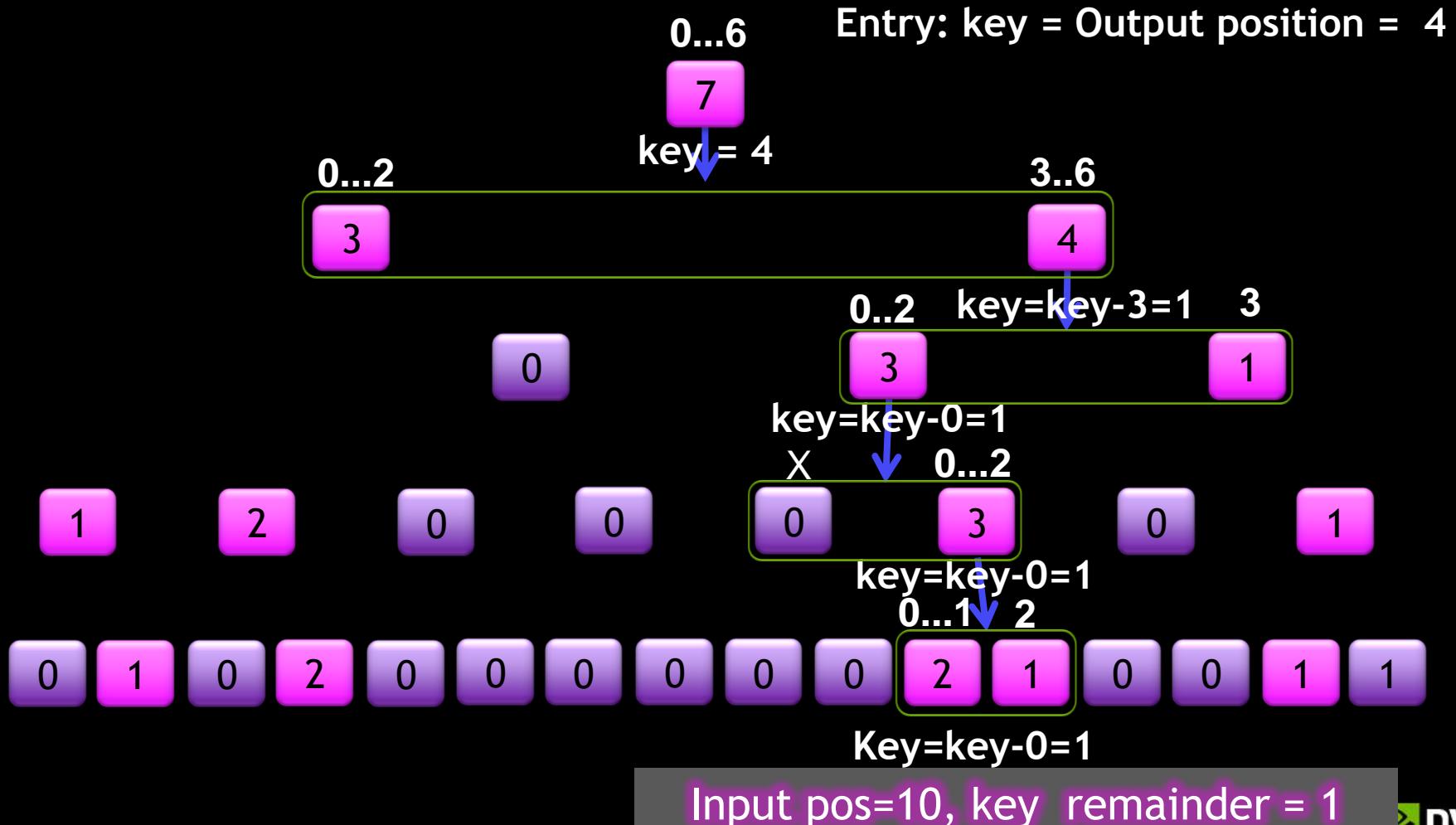
- Output size is known from top element of HP
- Allocate output
- Start one thread per *output element*
- Each thread knows its output index
- Now use HistoPyramid as  
search structure for finding corresponding input element

# HistoPyramid Traversal

- Each thread handles one output element
- *key* : variable, initially output index
- Binary Search through HP, from top-level to base-level
  - Reduction inputs x and y form *key ranges*  $[0, x)$  and  $[x, x+y)$
  - Choose fitting range for key
  - Subtract chosen range's start from key
- Note: For  $a_j > 1$ , several output threads will end up at same input element: key remainder is *index within this set*

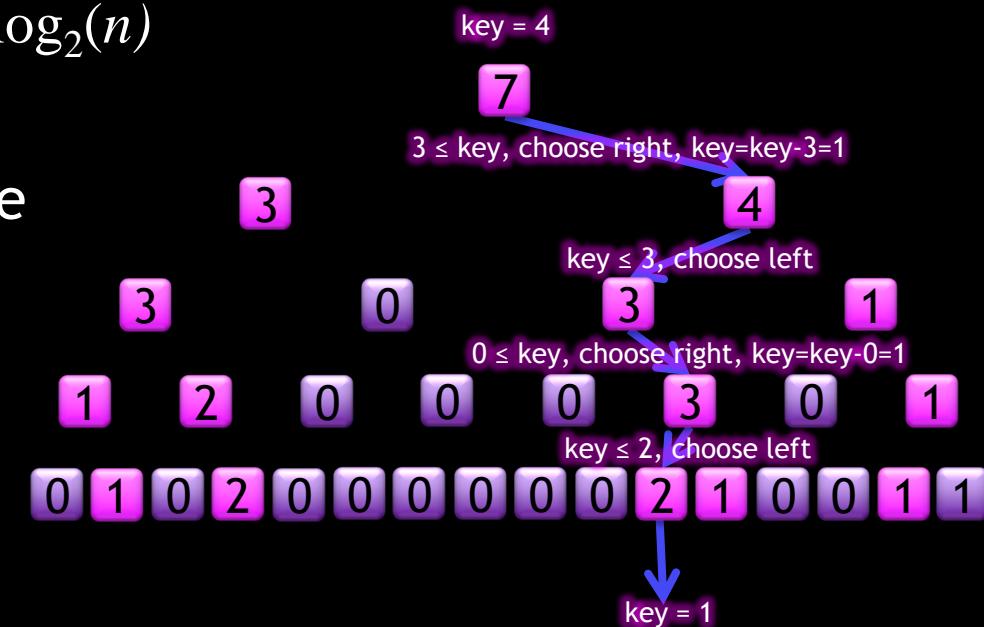


# HistoPyramid Traversal



# More observations on HP traversal

- Fully data-parallel algorithm (HP is read-only in traversal)
- Traversal steps/Data dependency:  $\log_2(n)$ 
  - Note: A pyramid has less latency
- Traversal path follows roughly a line
  - Adjacent output elements have very similar traversal paths
    - Good cache coherence
  - Large chunks of output elements have identical paths from top
    - Good for many-thread broadcast
- Some elements are never visited



# Optimization 1: Discard some partial sums

- Observation:

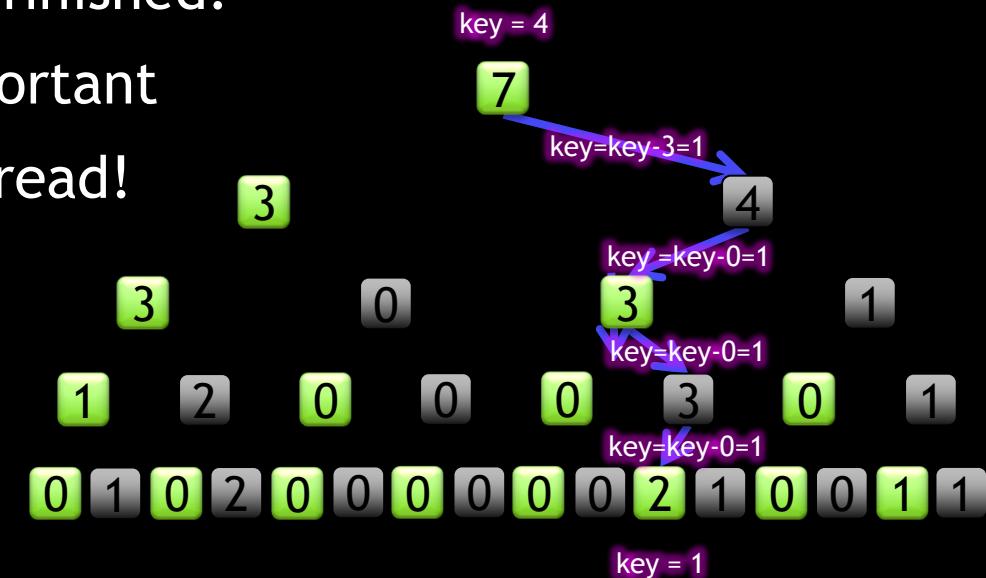
- In traversal, after build-up has finished:

- Only the **left** nodes are important
  - The right nodes needn't be read!

- We can **discard** all the right nodes

- Note: Number of all left nodes equals number of input elements

- Similarities to the Haar-transform!



# Optimization 2: k-to-1 reductions

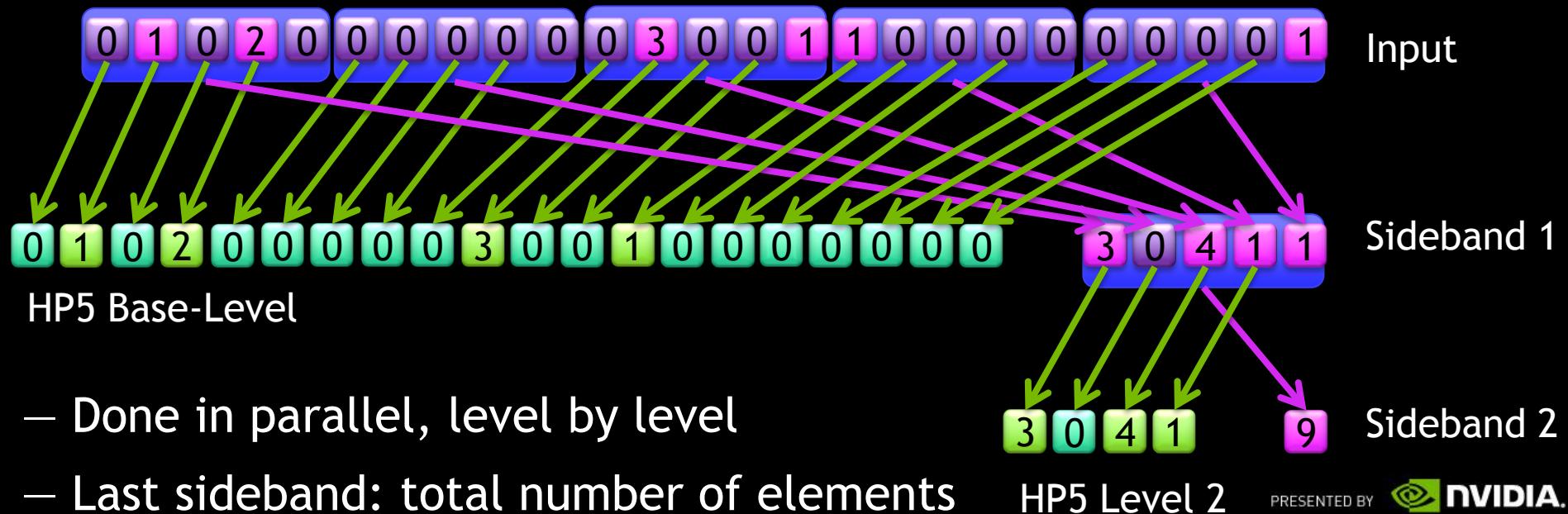
- Reduction does not have to be 2-to-1
- Example: 4-to-1 reduction is also possible
  - Fewer levels of reductions -> fewer levels of traversal :  $\log_4(n)$
  - Better for hardware (can fetch up to 4 values at once, reduce overall latency with fewer traversal steps)
  - HPMC from 2007 uses 4-to-1 reductions in 2D (texture mipmap-like)
    - Output extraction for consecutive elements follows space-filling curve in base level
    - Traversal: Adjacent HP levels accessed in mipmap-like fashion
    - **Excellent texture cache behaviour**

# HP5 (5-to-1 HistoPyramid)

- Combines two previous optimizations:
  - Buildup: Every reduction adds **five** elements into one output, **BUT**:
    - Only **four** of the reduction elements are stored!
    - Fifth reduction element goes to computational **sideband**
      - only acts as temporary data during reduction
- Traversal requires only first four elements
  - Fifth element is directly deducted during top-down path.
- Advantage of HP5:
  - **Less data storage**
  - **more efficient traversal**

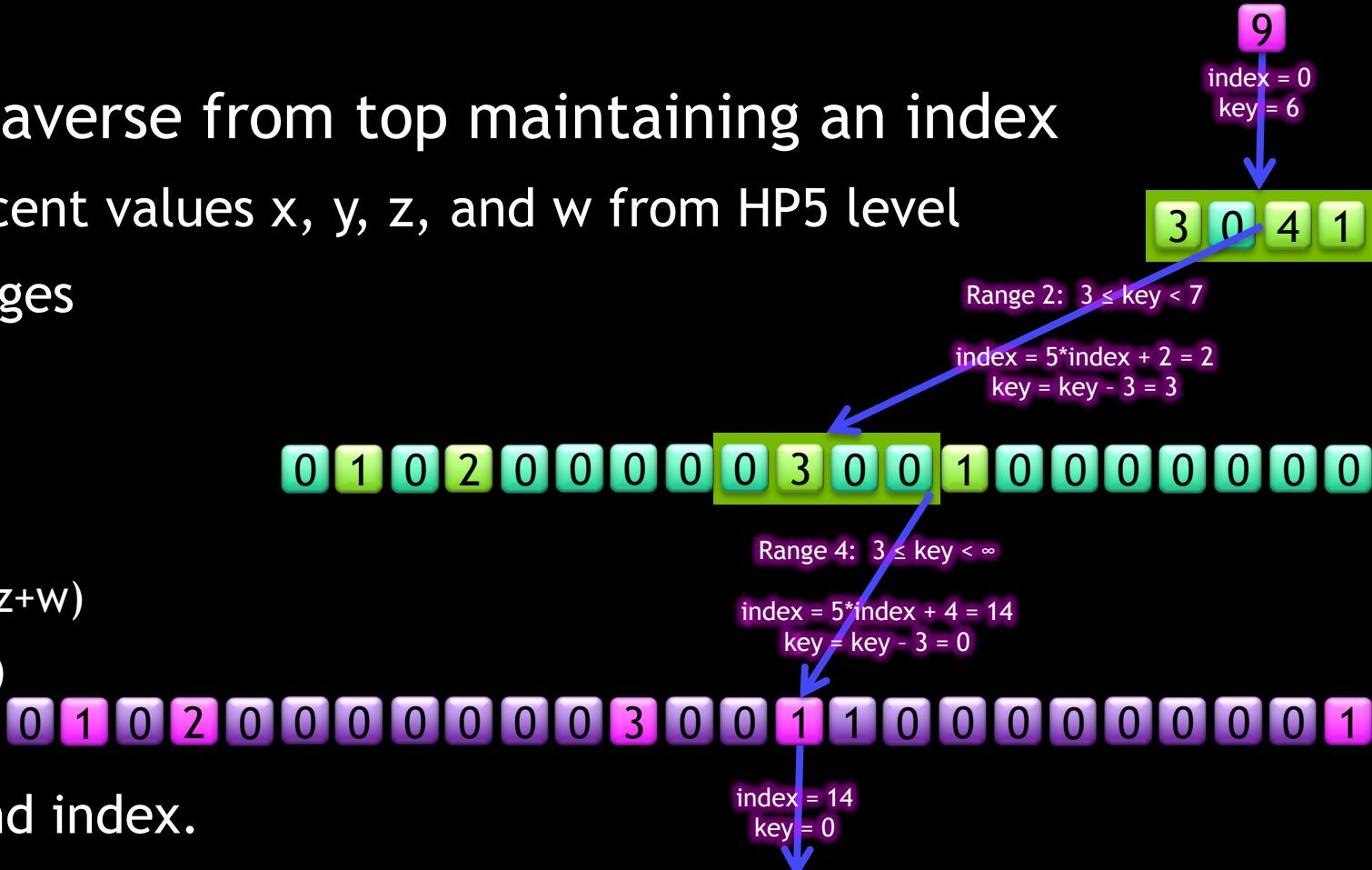
# The HP5 reduction

- For each group of 5 elements in input stream or sideband:
  - First 4 elements into HP5 level
  - The sum of the 5 elements into sideband



# The HP5 traversal

- Given a key, traverse from top maintaining an index
  - Fetch 4 adjacent values x, y, z, and w from HP5 level
  - Build key ranges
    - [0,x)
    - [x,x+y)
    - [x+y,x+y+z)
    - [x+y+z,x+y+z+w)
    - [x+y+z+w,  $\infty$ )
  - Check range, adjust key and index.



# HistoPyramid performance

- Data compaction: CUDA 3.2 SDK, Tesla C2050

2 million input elements, whereof N% retained	Scan	Atomic Ops	HP 4-to-1	HP 5-to-1
1% retained	0.70 ms	0.37 ms	0.34 ms	0.28 ms (2.5x)
10% retained	0.80 ms	3.04 ms	0.47 ms	0.38 ms (2.1x)
25% retained	0.81 ms	7.47 ms	0.63 ms	0.53 ms (1.53x)
50% retained	0.83 ms	14.89 ms	0.93 ms	0.81 ms (1.02x)
90% retained	0.85 ms	26.75 ms	1.40 ms	1.25 ms (0.60x)

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# Explanation: HistoPyramids vs. Scan

- Scan is **input-centric**

- Efficiently computes output offset for all input elements
- Uses one thread per input elements to write output (scatter)
- For **few relevant** input elements:
  - Redundantly computes output offsets for **all** input elements
  - Starts superfluous threads for **all**, and many irrelevant, input elements

- HistoPyramids is **output-centric**

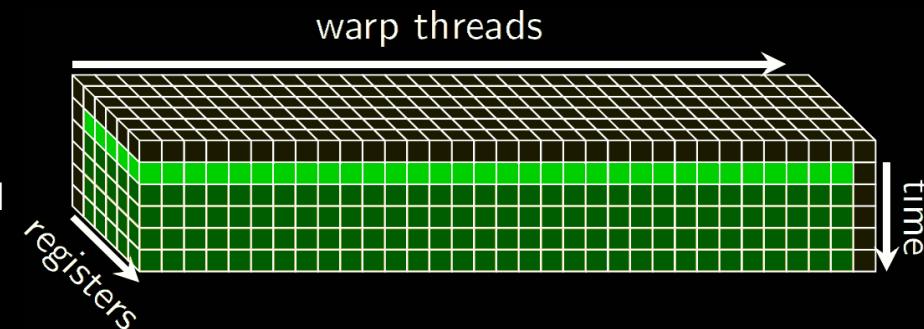
- Minimal amount of computations per input element
- Uses one thread per output element to write output (gather)
- **But:** requires HP traversal instead of a simple array look-up.

# HistoPyramid-based Marching Cubes

- Recall the 3-step subdivision of marching cubes:
  1. For each cell, determine case and find required # vertices
    - Embarrassingly parallel
    - Performed in **CUDA**
  2. Find total number of vertices and output-input index mapping
    - Build 5-to-1 HistoPyramid
    - Performed in **CUDA**
  3. For each vertex, calculate positions
    - Embarrassingly parallel
    - Performed directly in an **OpenGL vertex shader**

# Step 1: Cell MC Case and Vertex Count

- Adjacent MC cells share corners
  - Let a CUDA warp sweep through a 32x5x5 chunk of MC cells
    - Process XZ-slices slice by slice:
      - Check in/out state of 6 corners along Z, (1 state per cell)
      - exchange for cells processed by this thread (2 states per cell)
      - Pull results from previous slice, (4 states per cell)
      - Exchange results across warps (X-axis), (8 states per cell)
      - Use a 256-byte table to find number of vertices required for cell



- Recycles scalar field fetches and in-out classifications
  - 32x5x5 MC cases in 33x6x6 fetches = 1.5 fetches per cell

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# Step 2: HistoPyramid 5-way Reduction

- HistoPyramid built level by level, from bottom to top
  - Reduction kernel uses 160 threads (5 warps)
  - All **five warps** fetch input sideband element as uint's into shmem
    - Adjacent shared memory writes, **no bank conflicts**
  - Synchronize
  - One **single warp** sums and stores results in global mem
    - Each thread reads 5 adjacent elements from shared mem
      - Fetches with stride = 5, **no bank conflicts**
    - Output 4 elements to HistoPyramid Level ( as uint4's )
    - Store sum of the 5 elements in HistoPyramid sideband (as single uint's)

# Optimizing the HistoPyramid Reduction

- Reduce global mem traffic:
  - Sidebands are streamed through global mem between reductions
    - Combine **two reductions** into one kernel
      - Requires 800+160 uint's of shmem (3.8 K), **free of bank conflicts**
    - Combine **three reductions** into one kernel
      - Requires 800+800 uint's in shmem (6.3 K), **free of bank conflicts**
    - Combine **step 1** and **three reductions** into one kernel
      - Each warp processes  $32 \times 5 \times 5 = 800$  MC cells, 4000 per block
      - Shares shared mem with reduction, **no extra shared mem required**
- Reduce kernel invocation overhead
  - Build the apex of the HistoPyramid using a single kernel
    - Reduces the number of kernel invocations

# Step 3: Extract output vertices

- Performed **directly on the fly** in OpenGL vertex shader:
  - No input attributes
  - **gl\_VertexID** is used as **key** for HistoPyramid traversal
    - Terminates in corresponding MC cell
    - MC case gives template tessellation
    - Key remainder specifies lattice edge for vertex in template tessellation
  - Vertex position found by sampling scalar field at edge end points
- Uses OpenGL 4's **indirect draw**
  - Number of vertices to render fetched from buffer object
  - No CPU-GPU synchronization needed

# Results: MC Implementation Approaches

- NVIDIA Compute SDK's MC sample uses CUDPP
- HPMC library [<http://www.sintef.no/hpmc>]:  
HistoPyramids (4:1) in OpenGL GPGPU approach
- Our new development of HPMC uses CUDA HistoPyramid (5:1)

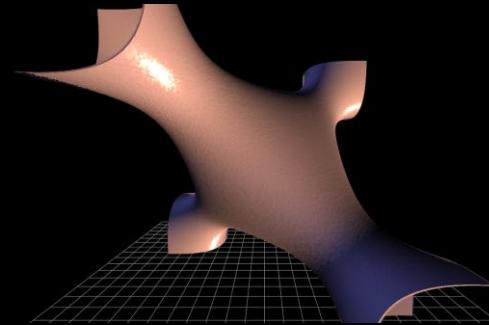
■ Key characteristics:

- Most often: 0 triangles per cell
- Maximally: 5 triangles per cell (=15 vertices)
- On average: 0.05 - 0.15 triangles per cell
  - Input (#cells) grows with cube of lattice grid resolution
  - Output (#triangles) grows with square of lattice grid resolution

# 256<sup>3</sup> 8bit performance (Tesla C2050)

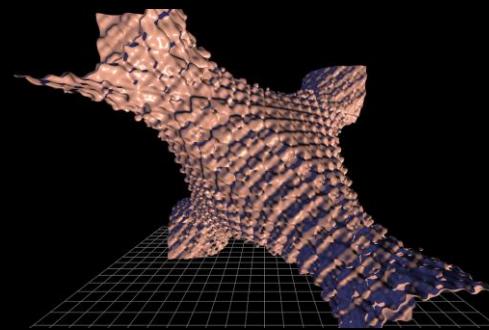
## Smooth Cayley (iso=0.5)

Triangles	445 522	(0.027 tris/cell)
NV SDK sample	72 fps	(1201 mvps)
OpenGL HP4MC	113 fps	(1868 mvps)
CUDA-OpenGL HP5MC	301 fps	(4985 mvps)
Speedup	<b>2.6x / 4.2x</b>	



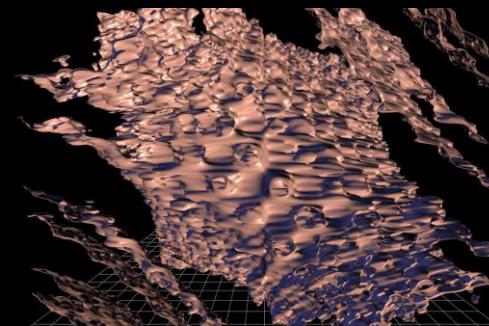
## Bumpy Cayley (iso=0.5)

Triangles	643 374	(0.039 tris/cell)
NV SDK sample	66 fps	(1098 mvps)
OpenGL HP4MC	102 fps	(1689 mvps)
CUDA-OpenGL HP5MC	242 fps	(4006 mvps)
Speedup	<b>2.4x / 3.6x</b>	



## Superbumpy and layered Cayley (iso=0.5)

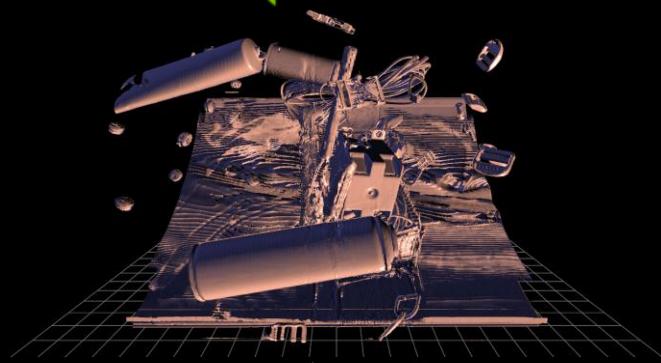
Triangles	3 036 608	(0.183 tris/cell)
NV SDK sample	34 fps	(571 mvps)
OpenGL HP4MC	47 fps	(774 mvps)
CUDA-OpenGL HP5MC	72 fps	(1199 mvps)
Speedup	<b>1.5x / 2.1x</b>	



# 512<sup>3</sup>-ish 16-bit performance (Tesla C2050)

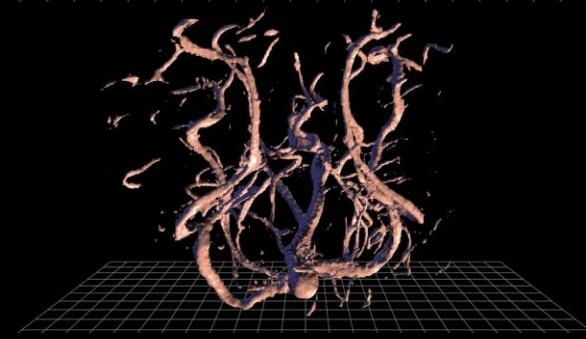
Backpack (iso=0.4) ([www.volvis.org](http://www.volvis.org))

Size	512x512x373	(187 mb)
Triangles	3 745 320	(0.039 tris/cell)
OpenGL HP4MC	13 fps	(1291 mvps)
CUDA-OpenGL HP5MC	43 fps	(4129 mvps)
Speedup	<b>3.2x</b>	



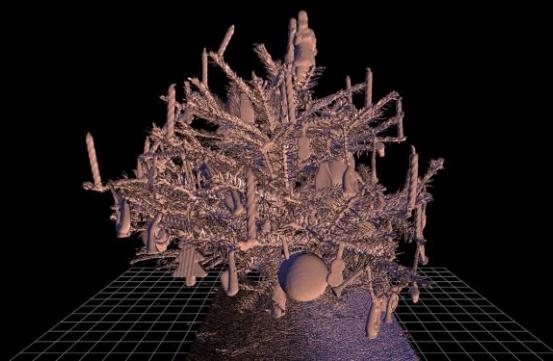
Head aneuysm (iso=0.4) ([www.volvis.org](http://www.volvis.org))

Size	512x512x512	(256 mb)
Triangles	583 610	(0.004 tris/cell)
OpenGL HP4MC	15 fps	(2034 mvps)
CUDA-OpenGL HP5MC	78 fps	(10399 mvps)
Speedup	<b>5.1x</b>	



Christmas tree (iso=0.05) (TU Wien)

Size	512x499x512	(250 mb)
Triangles	5 629 532	(0.043 tris/cell)
OpenGL HP4MC	10 fps	(1358 mvps)
CUDA-OpenGL HP5MC	28 fps	(3704 mvps)
Speedup	<b>2.7x</b>	



# CUHP5 Marching Cubes Showcase Video

[http://www.youtube.com/watch?v=WS95KjUS\\_Ww](http://www.youtube.com/watch?v=WS95KjUS_Ww)

# Summary

- Our SPH visualization approach is based on Marching Cubes
  - Requires high performance data compaction and expansion
  - Output size is considerably smaller than input size
- 5:1 HistoPyramid buildup and traversal
  - Optimizations: 5:1 instead of 4:1, leave out last leaf, shmem
  - Performance comparison for typical input-output ratio of 1-10%
- Implementing Marching Cubes
  - Implementation details
  - Performance
- Fastest Marching Cubes in the world ?

# CUHP5 Marching Cubes

Thank you!

Questions?

Chris Dyken <[christopher.dyken@sintef.no](mailto:christopher.dyken@sintef.no)>  
Gernot Ziegler <[gziegler@nvidia.com](mailto:gziegler@nvidia.com)>

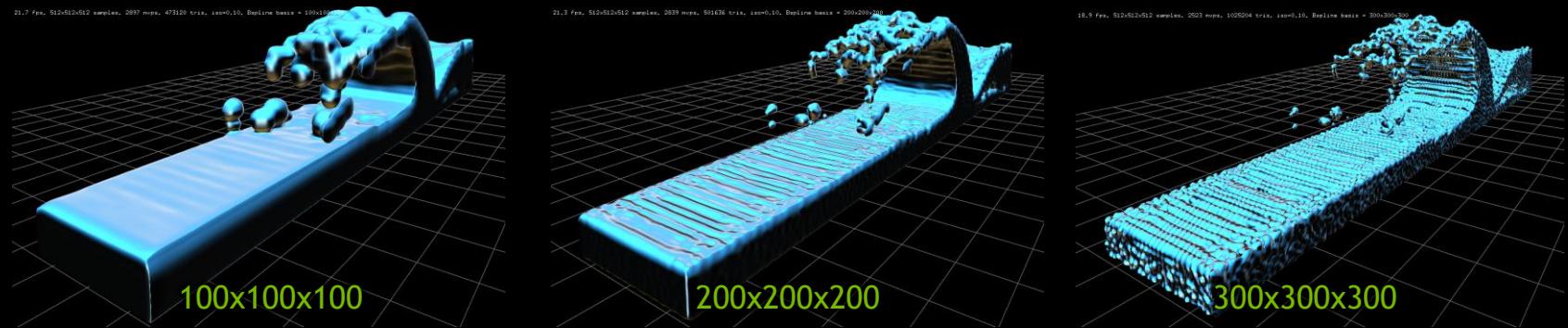
# CUHP5 Marching Cubes

BONUS SLIDES

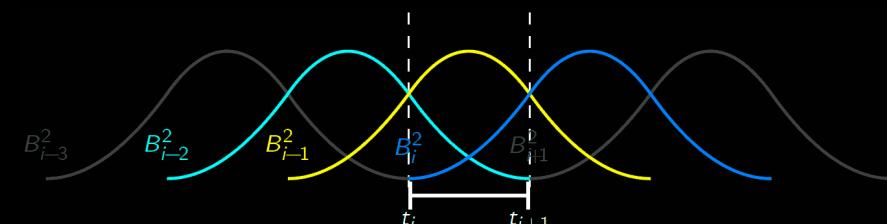


# Build a scalar field from the SPH nodes

- We approximate using a quadratic tensor-product B-spline
  - Simple and runs well on a GPU
  - Spline space size controls blurring versus detail



- A quasi-interpolant builds the spline
  - Contribution equals basis at position
    - Scatter contributions using atomic adds
    - No need to solve a linear system!



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