Computational Finance on the GPU

San Jose, California | September 30 – October 2, 2009

Objective

- Develop understanding of what "Computational Finance" means
- Learn who uses Computational Finance
  - Why do the users need it?
  - What do they do with the information?
- Take a look at some typical algorithms
- Consider the challenges and benefits of adoption

AGENDA

- Financial Instruments
- Who's Who
- Modeling
- Algorithms
- Adoption
- Summary

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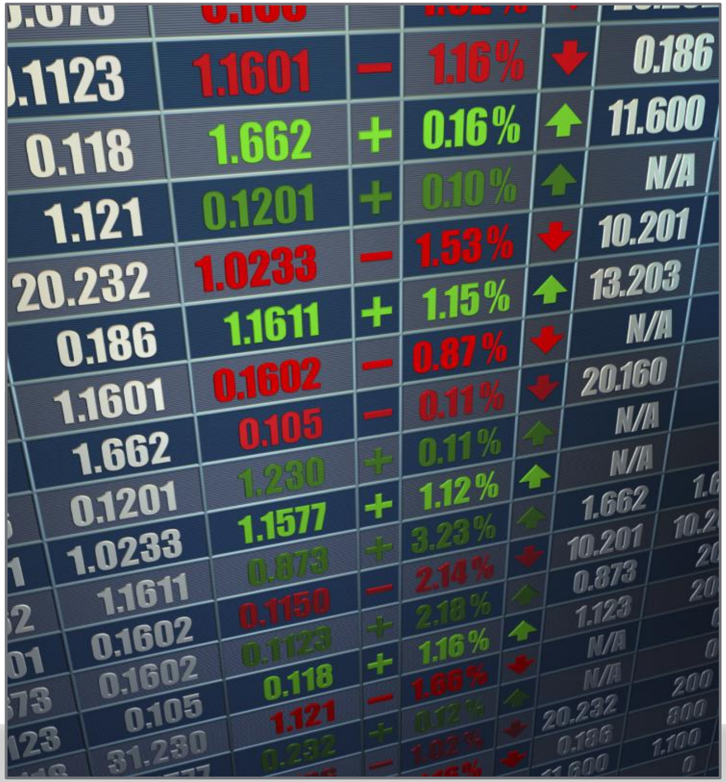
Financial Instruments

Cash Instruments	Derivatives
Equities	Exchange-traded
Commodities	Over-the-counter
Fixed Income	
Foreign Exchange	

Equities

- Share ownership
- Value determined by market
- Dividends

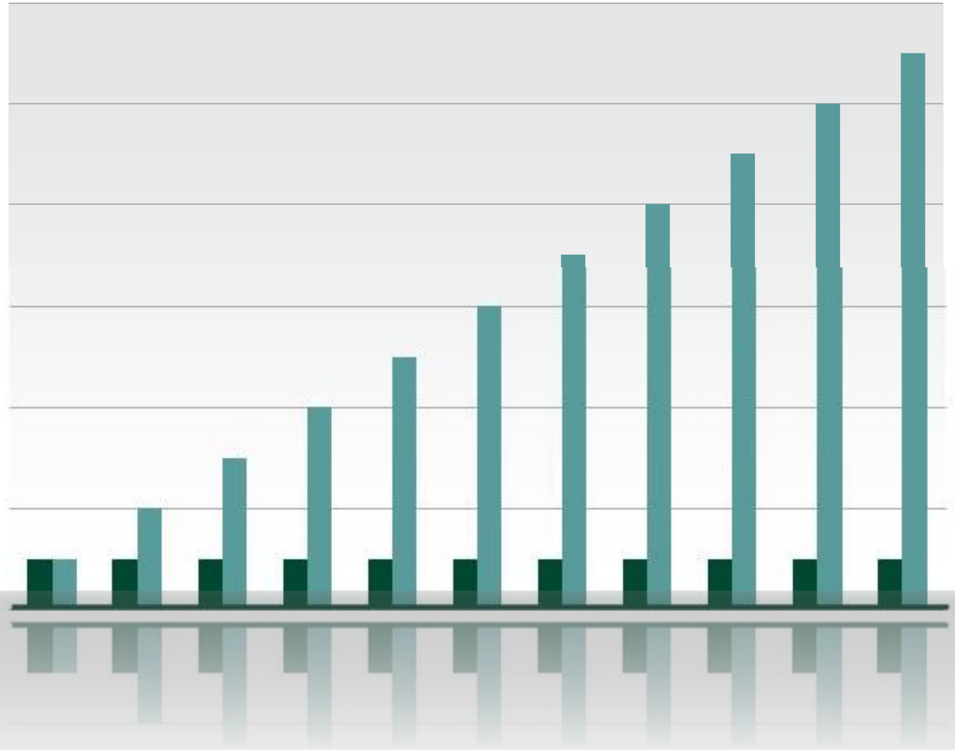


Commodities

- Raw resource
  - Agriculture: corn, rice
  - Livestock: pork bellies
  - Energy: oil, gas
  - Metals: precious, industrial
- Supply and demand

Fixed Income

- Also: Credit
- Loans and bonds
- Different rates according to duration



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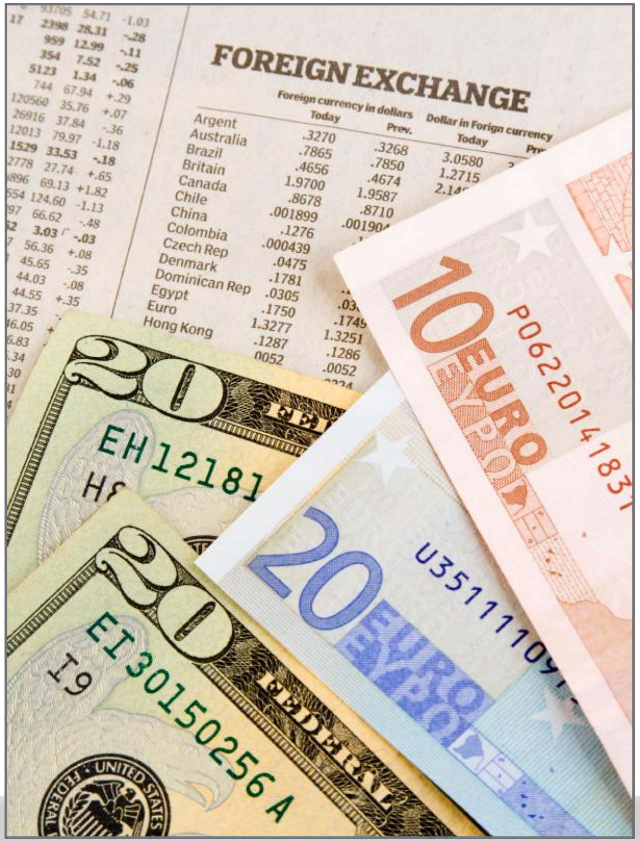
Foreign Exchange

- Also: Forex or FX
- Take advantage of changes in rates



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Derivatives

- Based on one or more underlying assets
  – Equities, FX, credit
- Many types of contract
  – Forwards and futures
  – Options
  – Swaps
- Exchange-traded or Over-the-Counter (OTC)

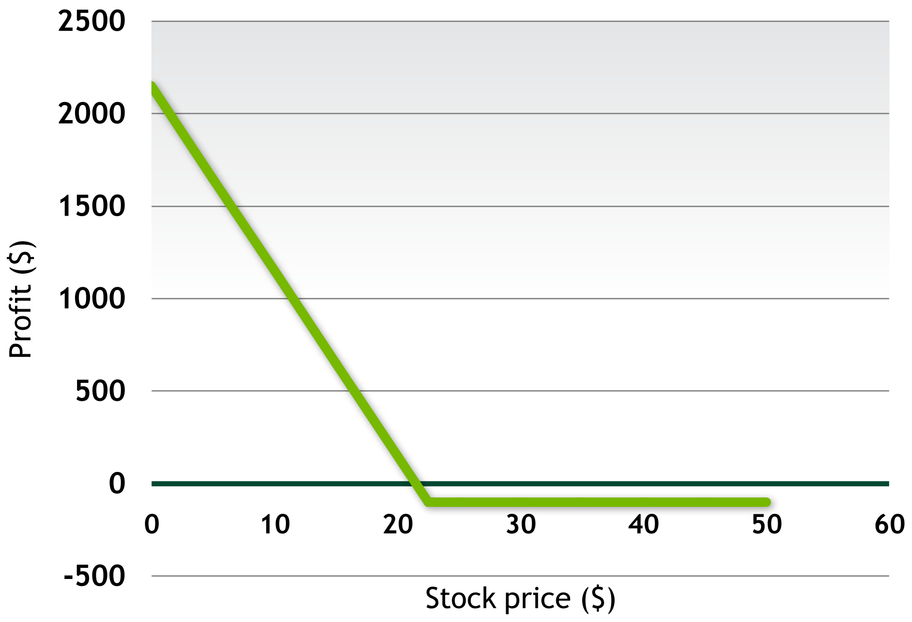
Example: Options

- Holder has the right to buy (*call*) or sell (*put*) the underlying asset
  - By a certain date
  - At a certain price

$$Payoff_{put} = \max(K - S, 0)$$

$$\text{where } K = \text{strike price}$$

$$S = \text{spot price}$$



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Traders

- Trading:
  - Standardised instruments
  - New (often complex) instruments
- Requires models:
  - Pricing
  - Prediction
  - Risk analysis

Traders | Backoffice | Quants | Developers

Backoffice

- Monitoring the banks exposure
- Model all trades
  - Value
  - Risk
- Value-at-Risk (VaR) required for regulation

Traders | **Backoffice** | Quants | Developers

Quants

- Develop and implement models for traders
- Develop independent models for validation
- Research
- Modelling exposure and capital

Traders | Backoffice | **Quants** | Developers

Developers

- Implement models from quants
- Integrate into larger applications
  - Interface to other models
  - Interface to database
  - Interface to user

Traders | Backoffice | Quants | **Developers**

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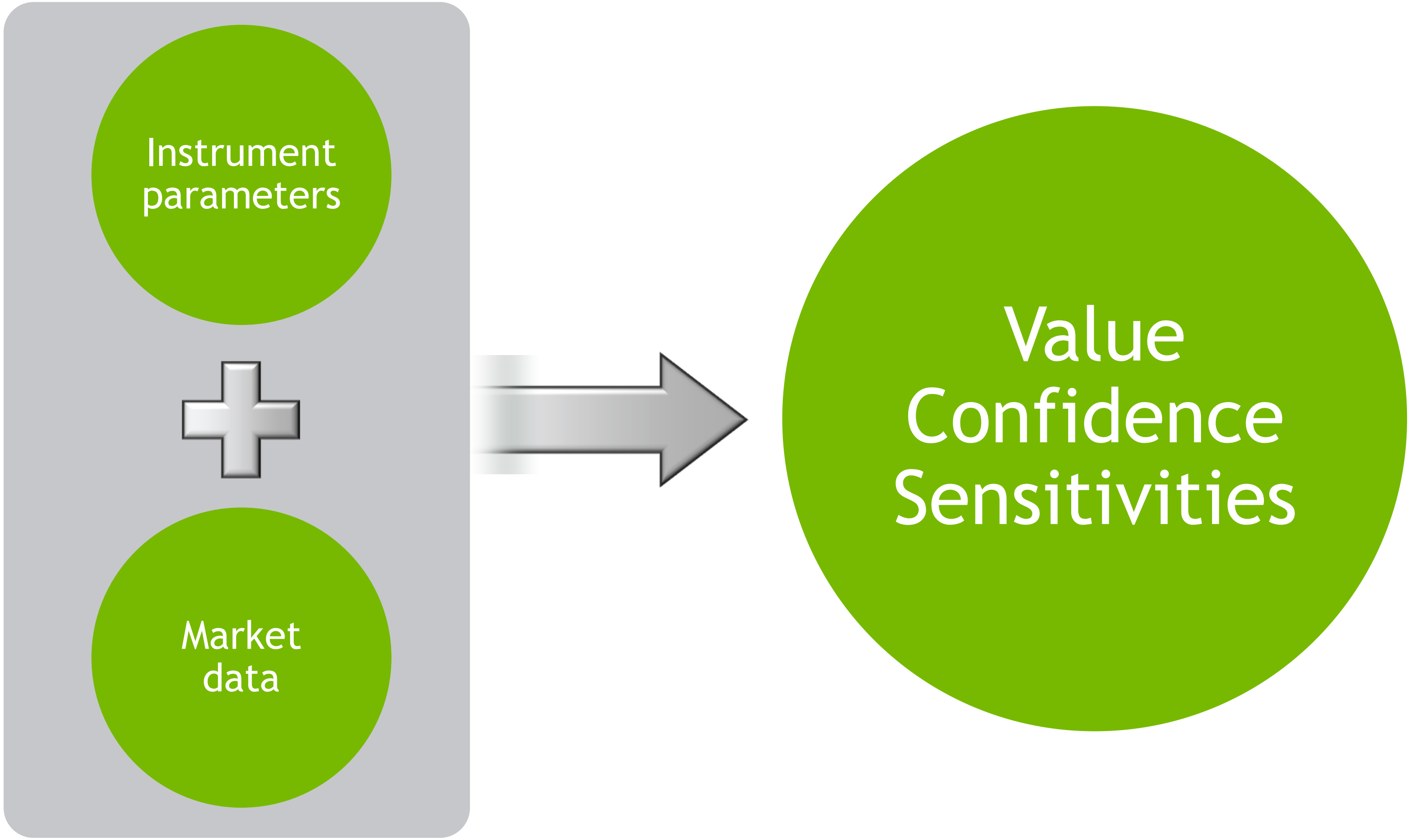
Inputs/Outputs



Traders

- Determine price for trade
  - Negotiate over the phone, require results fast
  - Minimize out-trades (errors)
- Run positions
  - Sensitivities allow trader to predict response to changes in underlying assets
  - Run often to allow trader to react quickly

Traders | Backoffice | Quants | Developers

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Backoffice

- Manage risk and capital reserves for regulation
  - Large runs can take days to complete
  - Accurate results allow greater control, and hence more trades
- Monitor traders' exposure
  - Run intra-day
- Greater accuracy requires longer run times

Traders | **Backoffice** | Quants | Developers

AGENDA

Analytic

- Some derivatives have an analytic solution

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf \quad \Rightarrow \quad \begin{cases} f = Ke^{-rT}\Phi(-d_2) - S_0\Phi(-d_1) \\ d_1 = \dfrac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\ d_2 = \dfrac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \end{cases}$$

Black-Scholes Formula

Price for European Put Option

- Compare analytical result with numerical result
  - Provides a Control Variate

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Binomial Trees

- Represent possible paths of stock
- Assumptions:
  - Stock has a probability *p* of moving up by a certain percentage *u*
  - Stock has a probability *(1-p)* of moving down by a certain percentage *d*

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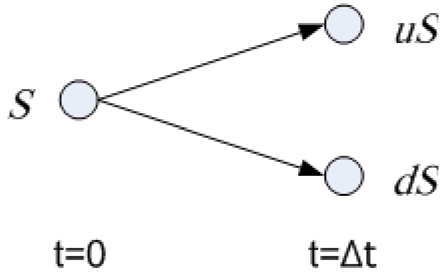
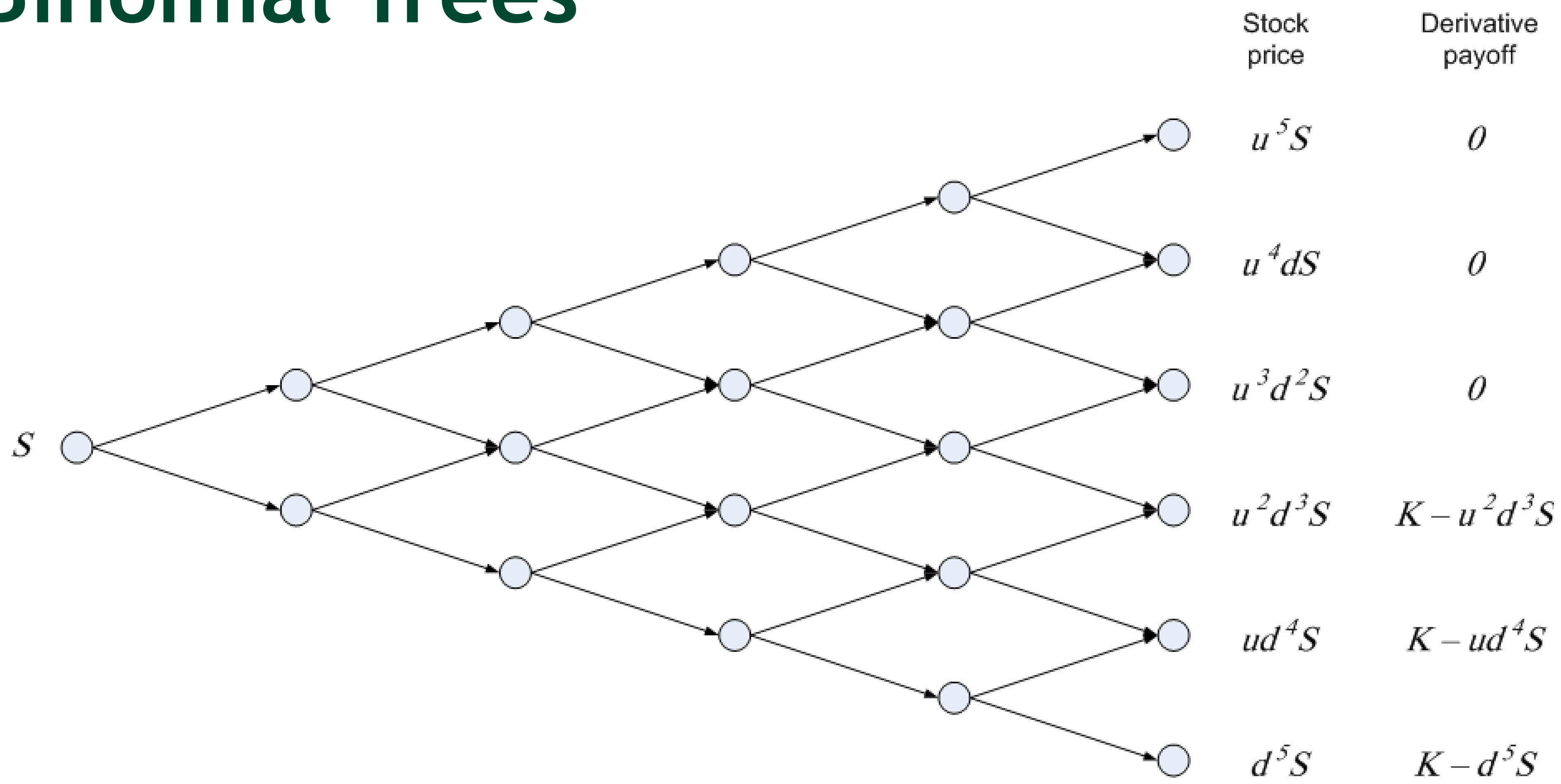
Binomial Trees

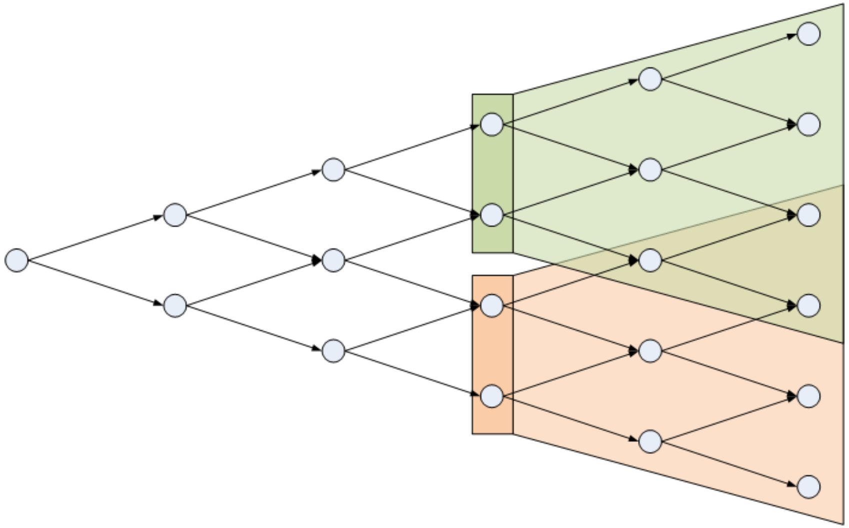


- Construct the tree
  - Create a branch for each time step
  - At each node the stock can either go up u% or down d%

Binomial Trees



Binomial Trees on the GPU

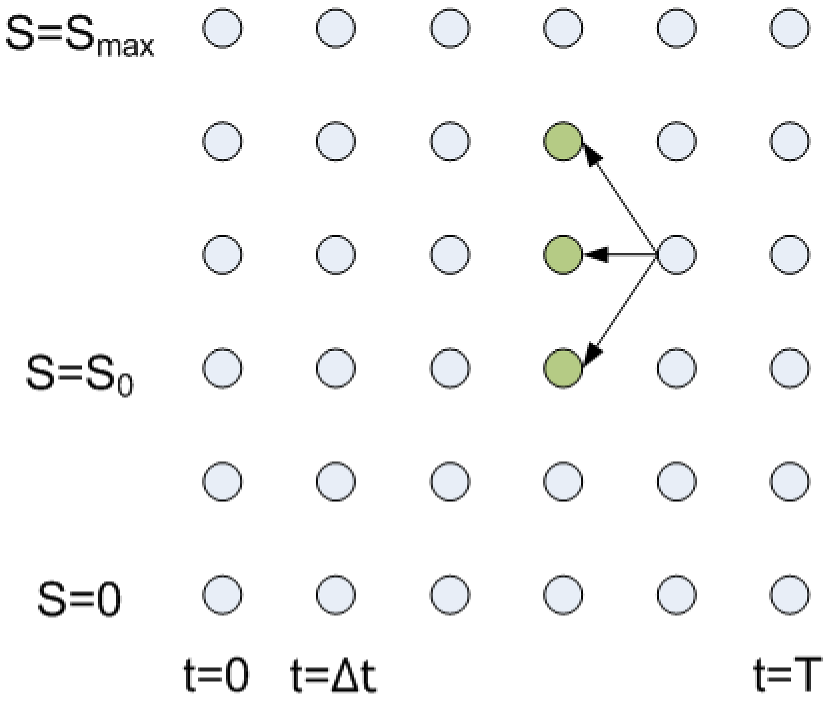


- Work backwards in time
  - Compute value at each node
- Partition the work across the SMs
  - Overlap input data
  - Fit input partition in smem

Finite Differences

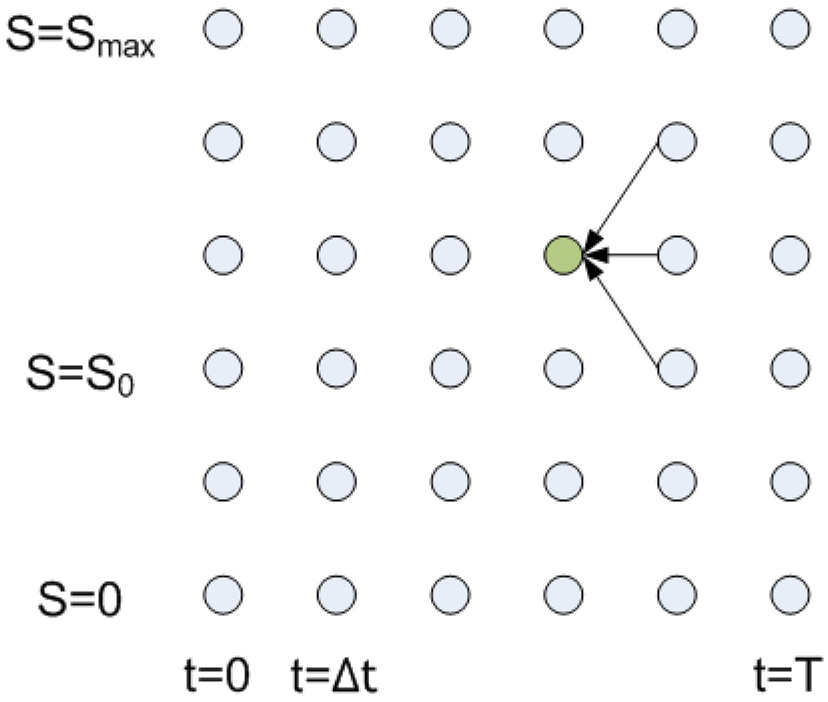
- Solve the Partial Differential Equation iteratively
  - Divide the life of the derivative into equal intervals of length Δt
  - Divide the range of stock prices $[0, S_{max}]$ into equal intervals of size ΔS
- Work backwards in time, compute the value at each node

Finite Differences - Implicit



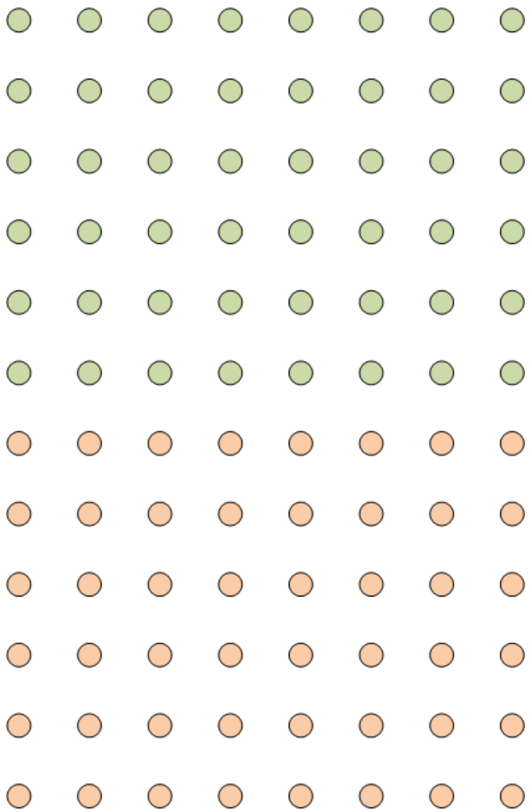
- Relationship:
  - Three values at *t* and one value at $t + \Delta t$
- Always converges to solution
- Requires solving simultaneous equations

Finite Differences - Explicit



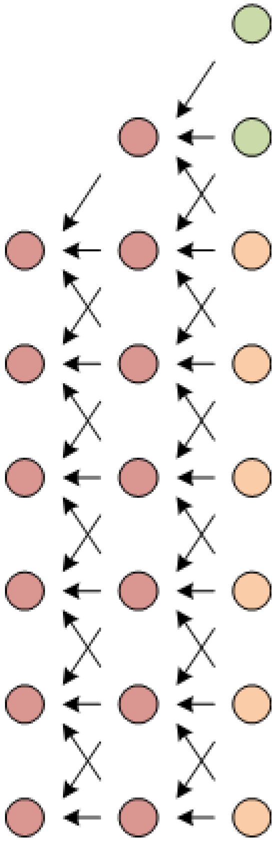
- Relationship:
  - One value at t and three values at $t + \Delta t$
- Compute nodes in parallel
- Can diverge from solution

Explicit Finite Differences on the GPU



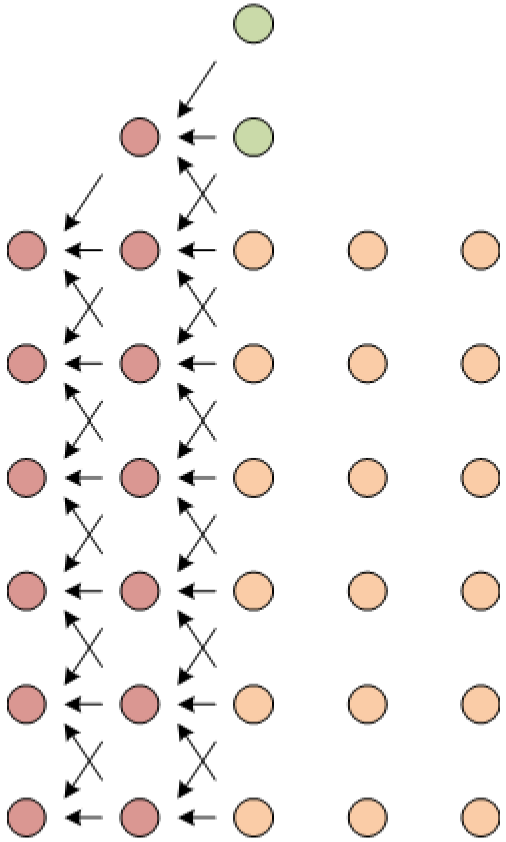
- Partition the grid across the SMs

Explicit Finite Differences on the GPU



- Partition the grid across the SMs
- Each SM requires a "halo"
  - Halo size determines how many time steps in batch

Explicit Finite Differences on the GPU

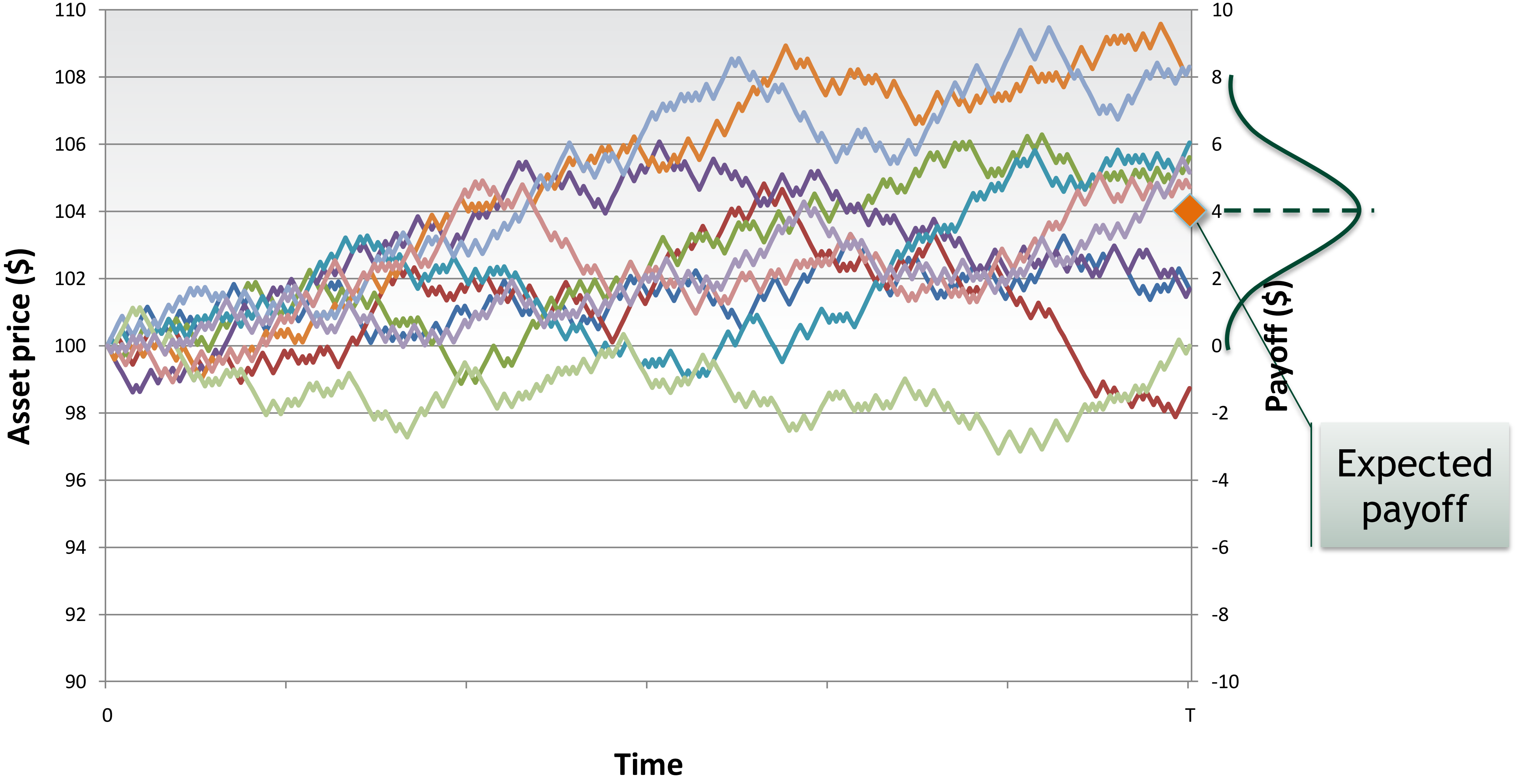


- Partition the grid across the SMs
- Each SM requires a "halo"
  - Halo size determines how many time steps in batch
  - After each batch, distribute new halos

Monte Carlo

- Sample a random walk for the asset(s)
- Calculate the payoff of the derivative
- Repeat to get many sample payoff values
- Calculate the mean payoff

Monte Carlo


110
108
106
104
102
100
98
96
94
92
90
10
8
6
4
2
0
-2
-4
-6
-8
-10
Asset price ($)
Payoff ($)
0
T
Time
Expected payoff

Monte Carlo on the GPU

Generate Random Numbers

- Distribution
- Covariance

→

Compute payoff for each path

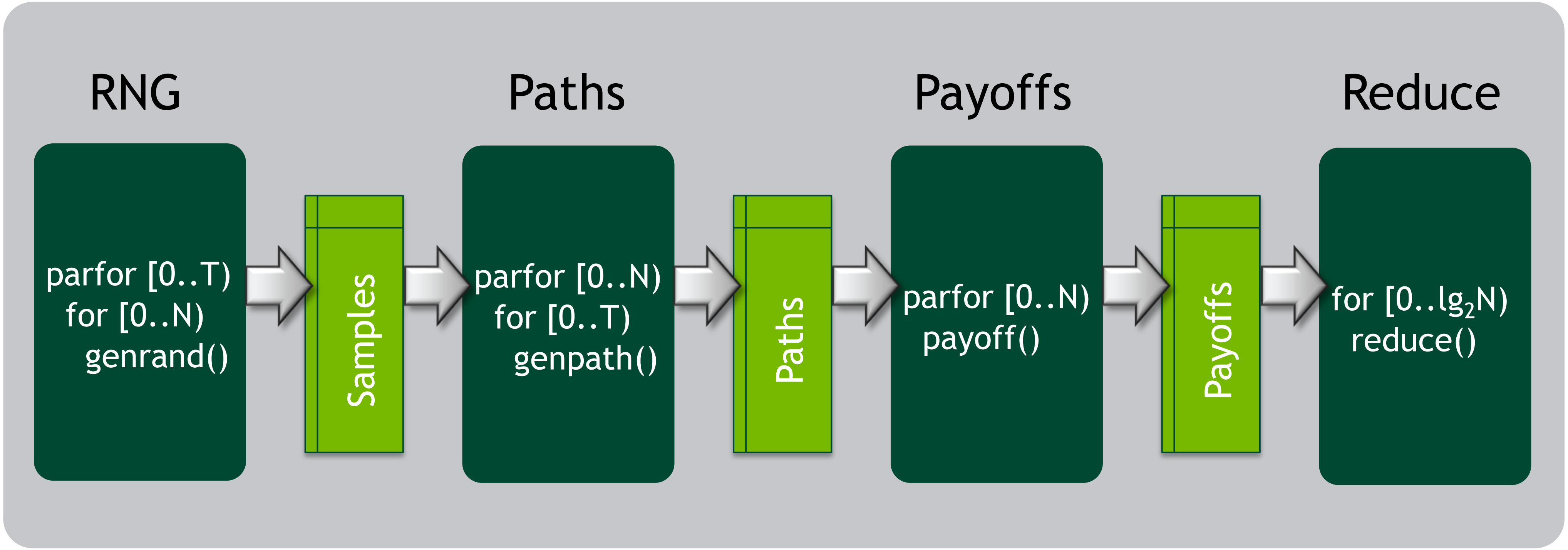
- Scenarios are independent

→

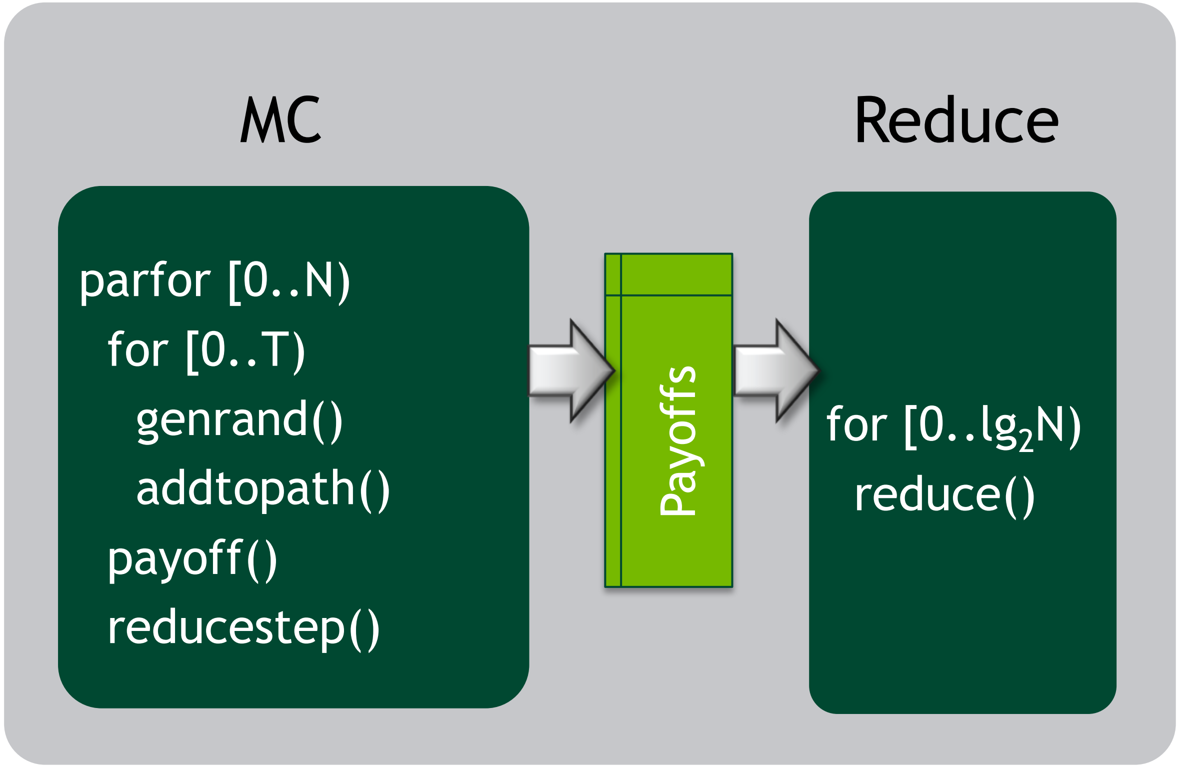
Compute statistics

- Reduction

Monte Carlo - Multiple Kernels



Monte Carlo - Single Kernel



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Software legacy

- Millions of lines of code
- Complex relationships between code blocks
  - E.g. Primary algorithm generates paths which are reused in multiple payoff models
- Hundreds of man-years of work
- Significant refactoring of application required
  - Support/feed the parallelized algorithms

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Education

- Parallel programming is paradigm shift
  - Quants are starting to rethink algorithms
  - Designing or reusing different algorithms/strategies
- Reuse of libraries and frameworks
  - Concentrate on the core algorithm
  - Increasing number of libraries for random numbers, linear algebra, reduction etc.

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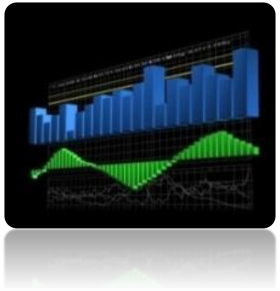
Case Study: Equity Derivatives



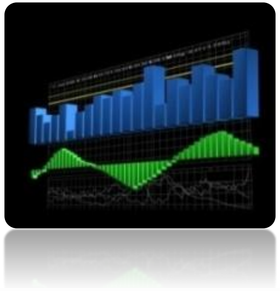
BNP PARIBAS

15	15x Faster	1
2 Tesla S1070	16x Less Space	500 CPU Cores
$24 K	10x Lower Cost	$250 K
2.8 KWatts	13x Lower Power	37.5 KWatts

Source: BNP Paribas, March 4, 2009

Case Study: Equity Derivatives

No need to compromise



BNP PARIBAS

15	15x Faster	1
2 Tesla S1070	16x Less Space	500 CPU Cores
$24 K	10x Lower Cost	$250 K
	13x Lower Power	37.5 kWatts

190x Lower Power in Total

Source: BNP Paribas, March 4, 2009

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Case Study: Real-time Options

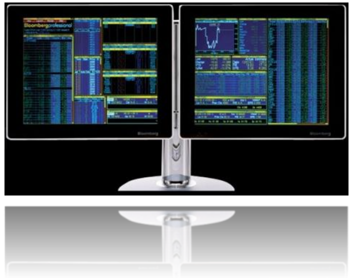
Hanweck Associates, LLC

1	Same Performance	1
3 Tesla S870	9x Less Space	600 CPU Cores
$42 K	6x Lower Cost	$262 K
$140 K	9x Lower Annual Cost	$1,200 K

Figures assume:

- NVIDIA Tesla S870s with one 8-core host server per unit
- CPUs are 8-core blade servers; 10 blades per 7U
- $1,800/U/month rack and power charges, 5-year depreciation

Case Study: Security Pricing



Bloomberg

2 hours	**8x Faster**	16 hours
48 Tesla S1070	**10x Less Space**	8000 CPU Cores

Source: Wall Street & Technology, September 24, 2009

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Conclusion

- Computers are used to model financial instruments for price, sensitivity and risk
- Algorithms include Finite Differences and Monte Carlo
- Parallelising algorithms requires structural support from the application
  - Benefit is substantial on all measures
  - GPUs are transforming the industry
- Opportunities for algorithmic development

Resources

- GTC presentations
  - Finance presentations, Thursday from 2pm, Atherton Room
  - 3D Finite Differences on GPU, Friday 10.30am, Empire Room
  - Tridiagonal solvers on GPU, Friday 2pm, Atherton Room
- SDK examples
  - binomialOptions, 3DFD, MonteCarlo/MonteCarloMultiGPU
- NVIDIA finance page (links to online resources)
  - http://www.nvidia.com/object/computational_finance.html