



GPU Accelerated Solvers for ODEs Describing Cardiac Membrane Equations

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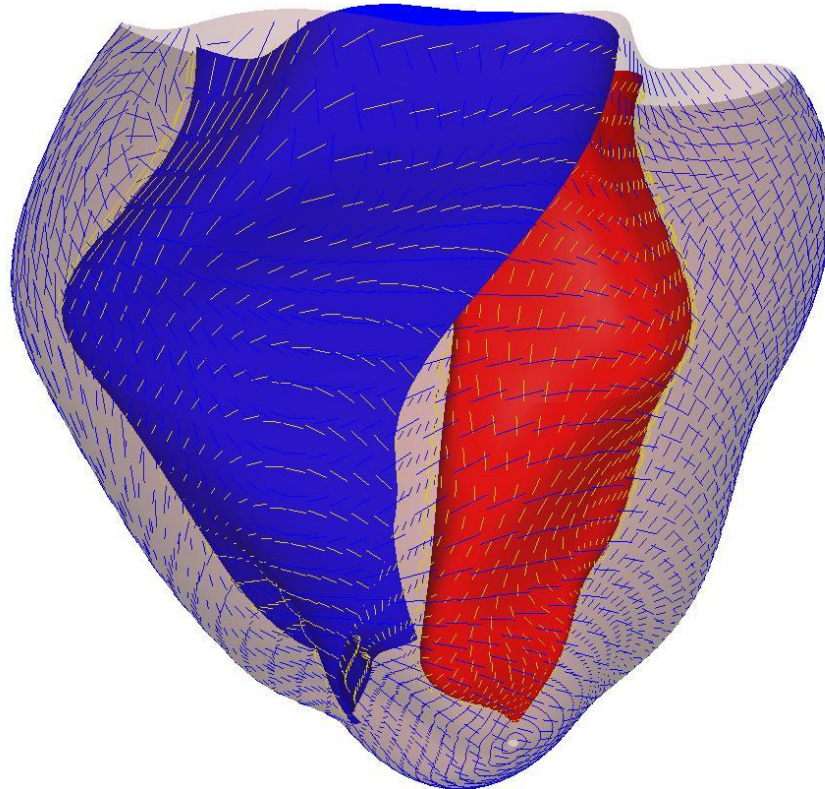
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What is heart modeling?

- Bioengineer authors a mathematical model to describe certain aspects of the heart's behavior
- Develop and perform computer simulation of mathematical model

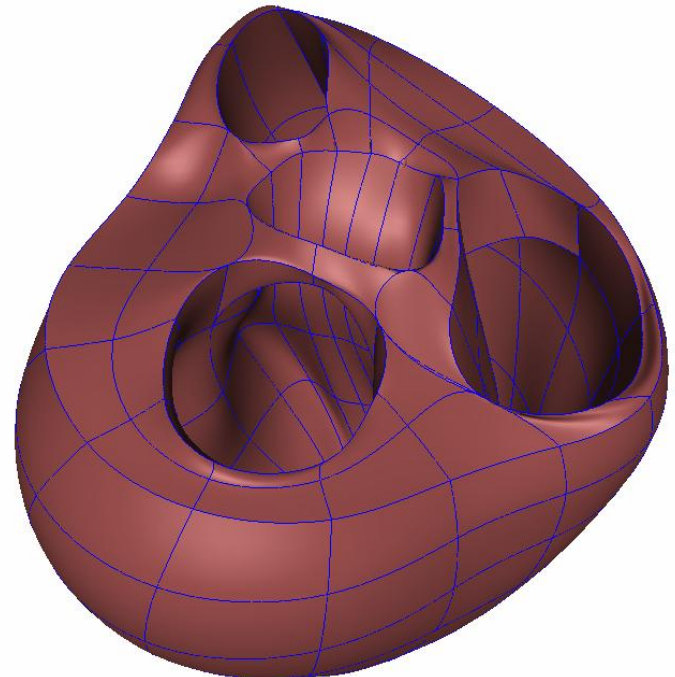
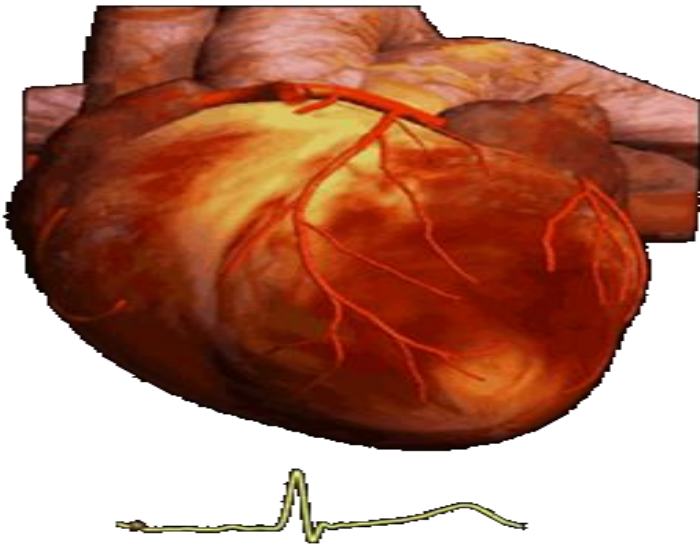


Why do heart modeling?

- Heart disease is the leading cause of death
- Basic Science
 - There still quite a bit about the heart we don't understand
- Clinical and therapeutic applications
 - Targeting ablation therapy for atrial arrhythmias
 - Defibrillator design
 - Cardiac resynchronization therapy (CRT)

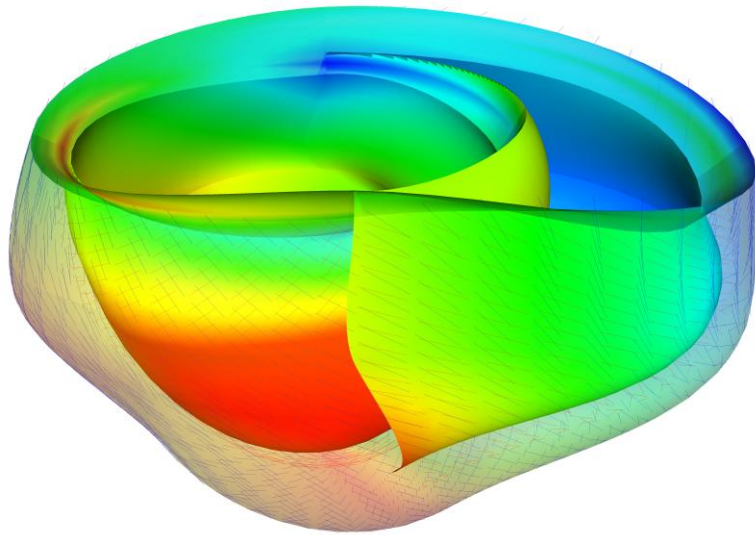
How do we do heart modeling?

- Finite Elements Method
 - Discretizing a continuous domain into a set of discrete sub-domains.
 - Useful for bridges, airplanes, buildings, and even hearts

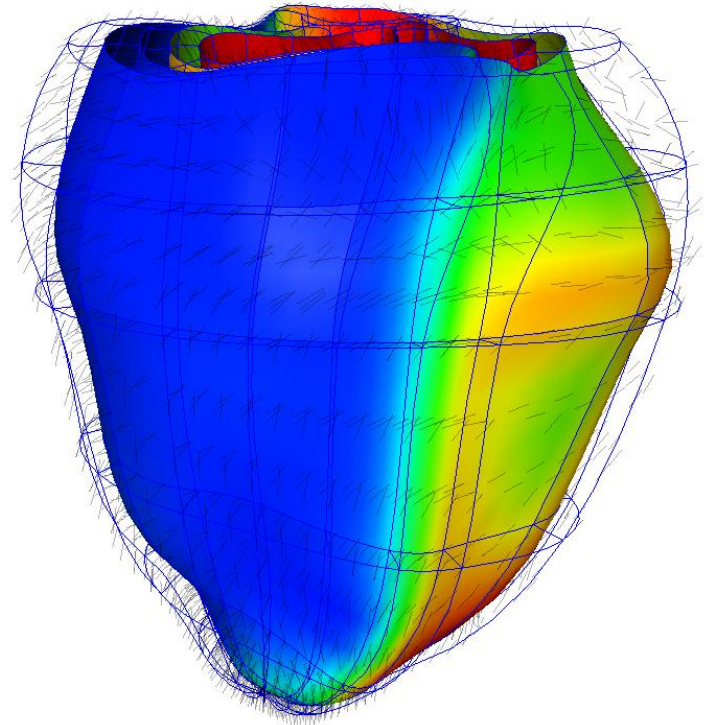


What kind of heart modeling simulations can we perform?

- Biomechanics

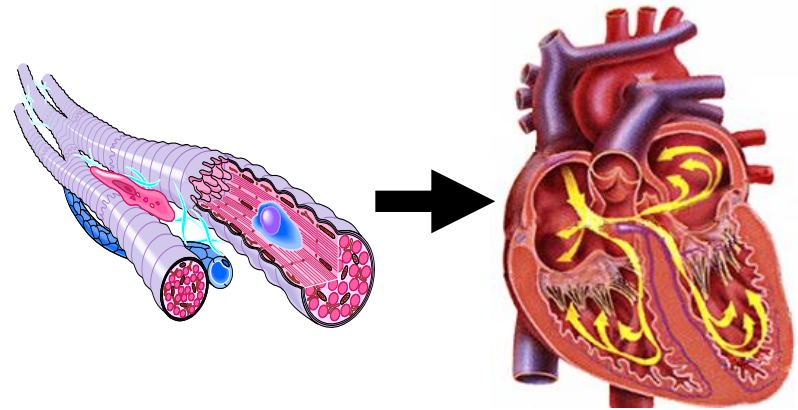


- Electrophysiology



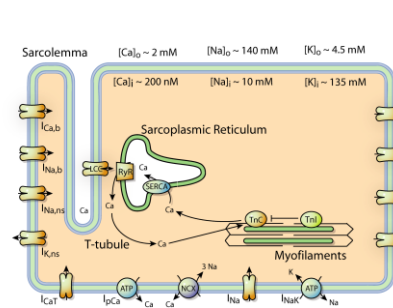
What are the bottlenecks?

- ODEs = 98.7%
- PDEs = 1.3%

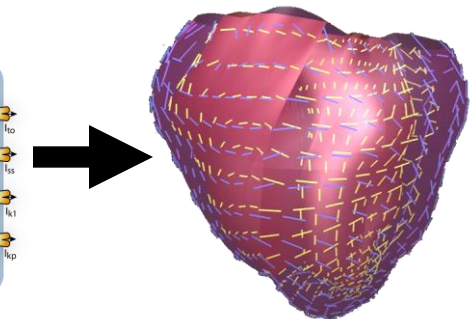


whole cell

ventricles



**Whole cell
ionic model
(odes)**



**3-D mono- or
bidomain finite
element model
(pdes)₆**

How can we optimize our ODEs?

- Multithreaded OpenMP?
- MPI Cluster?
- GPU?

What about the GPU?

- Benefits
 - ODEs have thousands or even millions of data parallel threads
 - nVidia GPUs have peak GFLOP rates that are 1 to 2 orders of magnitude higher than a quad core processor
 - CUDA provides straightforward implementation

CUDA Acceleration Challenges

- Performance
 - Memory access
 - Thread Divergence
- Accuracy
 - Precision
 - CUDA math library
- CUDA Code Generation
- CUDA ODE solver
 - Too stiff for Forward Euler or Explicit Runge-Kutta
 - Capture processes on a wider range of time scales
 - Stiff if step size required for stability \ll required for accuracy
 - Simplified Single Iteration Backwards Euler solver
 - Adaptive solvers lead to thread divergence

The precision problem

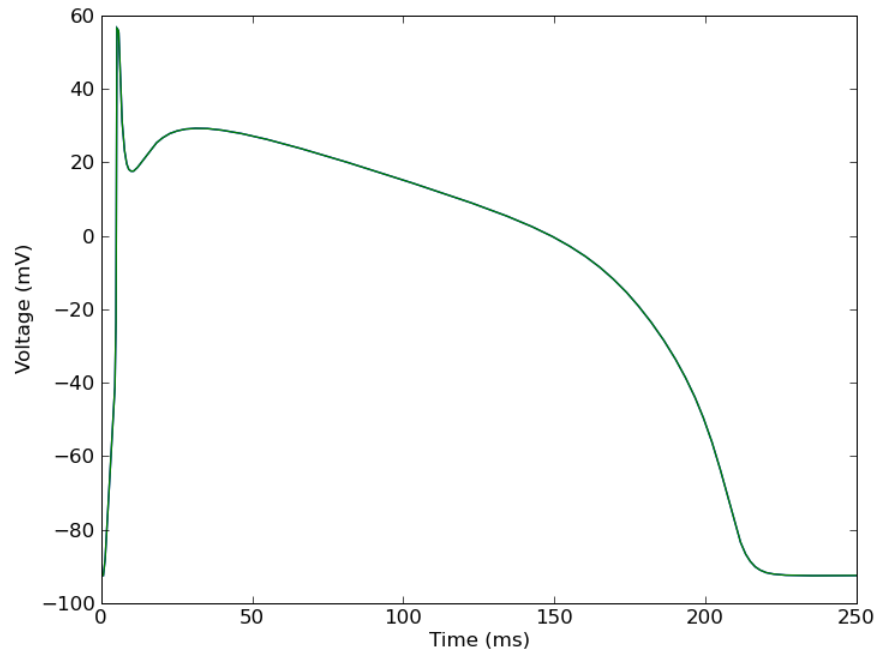
- Singularities at magic numbers
 - Nothing biologically significant about these numbers
- Must identify these problems and correct them
 - Reformulate equation
 - Add a workaround

$$a = \frac{b}{e^{\text{voltage}} - 1}$$

$$\lim_{v \rightarrow 0} \frac{1}{e^v - 1} = \infty$$

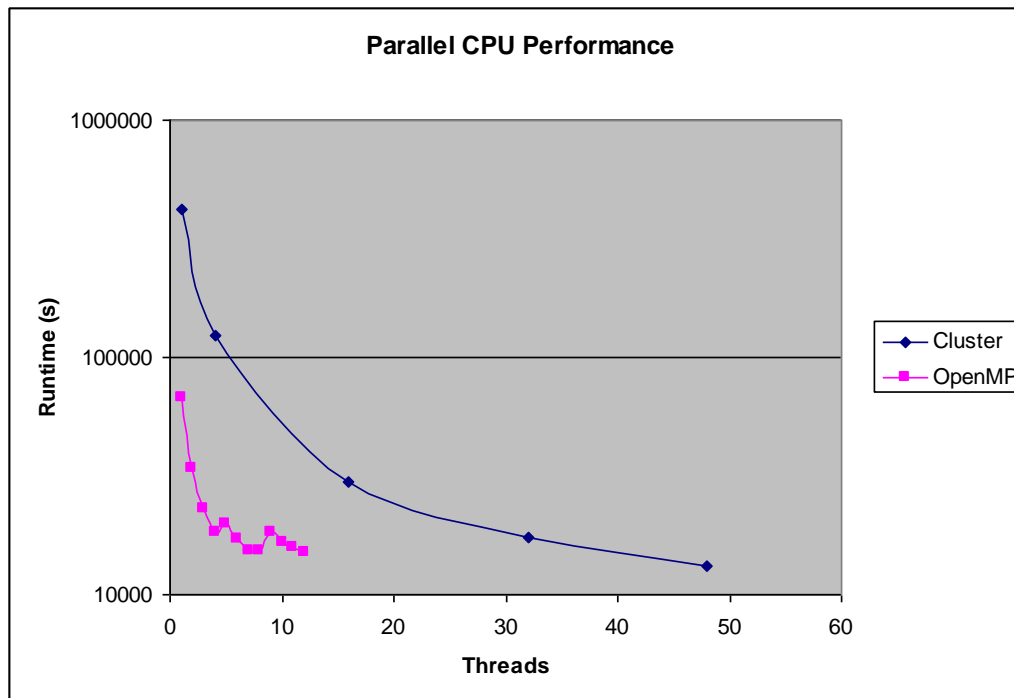
The Flaim Cell Model

- Developed by Sarah Flaim in 2006
- 87 ODEs
- Dog heart



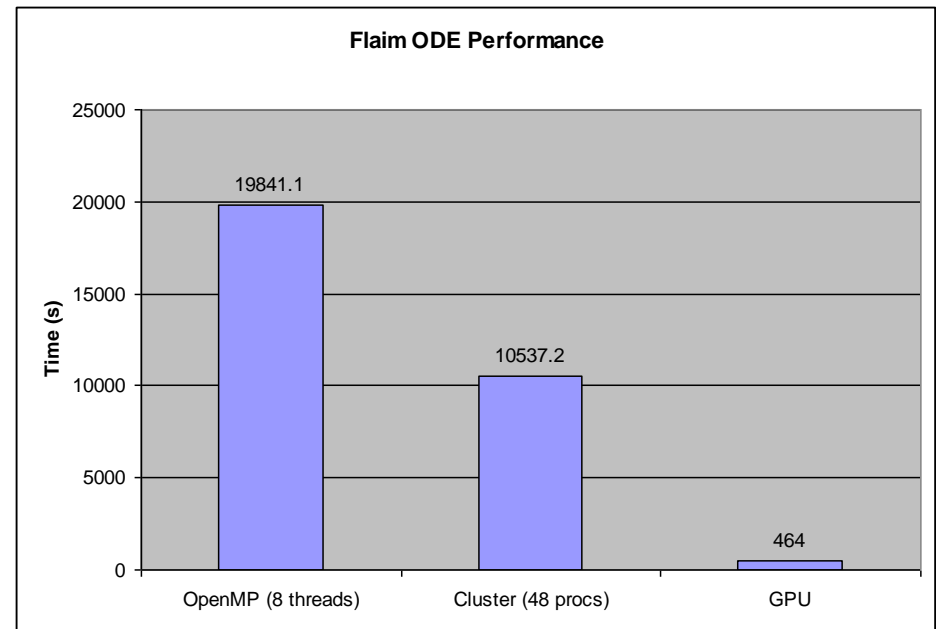
Conventional Acceleration of the Flaim Model

- Desktop (assembled in 2009)
 - Intel i7 quad core processor at 2.93GHz with 12 GB ram
 - OpenMP
- Cluster (assembled in 2007)
 - AMD Opteron 2216 dual core processors 2.4 GHz
 - Each of the 66 compute nodes has 4 cores, 4 GB of RAM.
 - MPI

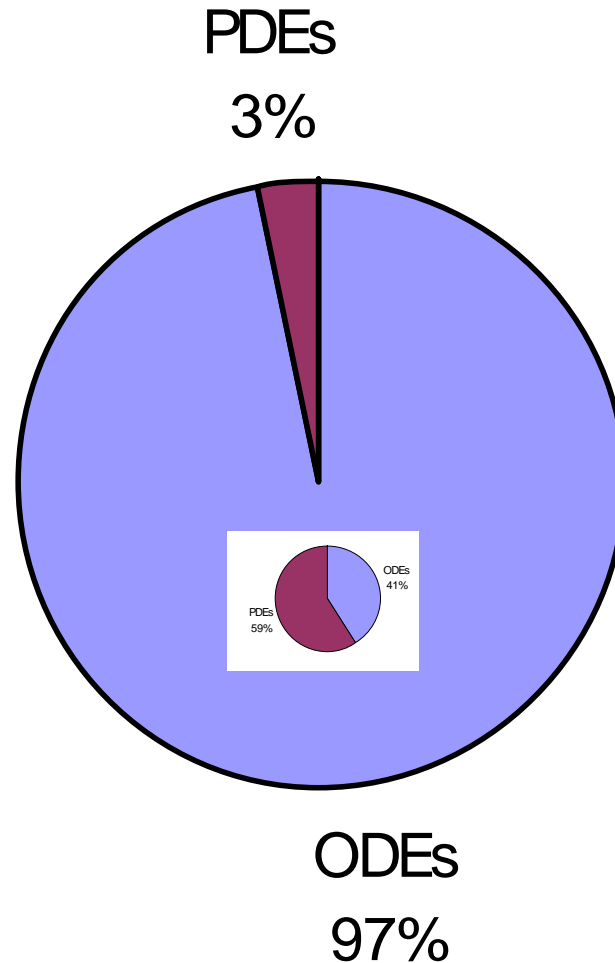


GPU Acceleration of the Flaim Model

- nVidia GTX 295
- Approximately 1 beat (300ms)
- Speedup Over OpenMP: **31x**
 - 5.5 hours → 7.7 minutes
- Speedup Over Cluster: **23x**
 - 3 hours → 7.7 minutes
- Single Precision Error: 0.85%
- Excellent GPU Occupancy
 - 8 blocks of 64 threads



Electrophysiology Bottleneck



Cell Model Results

	Year	ODEs	Registers and local memory (words)	Speedup over OpenMP	Runtime CUDA (s)	Runtime OpenMP (s)	% Error
FitzHugh-Nagumo	1961	2	16/0	21.9x	7.8	171	0.06%
Beeler-Reuter	1977	8	32/0	172.2x	16.2	2789.1	2.23%
Puglisi	2001	18	32/60	63.7x	101.4	6461.1	1.34%
Flaim	2006	87	32/392	31.3x	462	14446.2	0.85%

Conclusions

- Single precision must be handled carefully, but is acceptable for many applications
- Maintaining high occupancy can be very important
- Went against some conventional wisdom
 - Spilling to local memory not so bad
 - Shared memory limited

The Future

- GPU is changing the way we think about heart modeling
 - A simulation that took 3 hours on our *cluster* can now be performed in ~7 minutes on a *desktop!*
 - Larger problems are now feasible
 - Essential component in hospital of the future?
 - Patient specific modeling
 - Much broader “parameter sweeps” now feasible
 - Interactive (e.g. real time) simulations are within reach for small problems
 - Save lives

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Questions?