Objective: Investigate the performance and scalability of a multigrid pressure Poisson equation solver running on a GPU cluster.

Method: A parallel 3D multigrid pressure solver was written for GIN3D, a 3D incompressible Navier-Stokes flow solver which runs on GPU clusters. Tests were performed using the well-known lid-driven cavity and natural convection in a cavity problems.

Solver: For pressure Poisson equation in incompressible Navier-Stokes flow solver.

Typically consumes a major portion of the time.

Simple iterative solvers such as Jacobi and Gauss-Seidel can run very efficiently on the GPU, but solutions converge very slowly.

Multigrid methods can converge rapidly and allow convergence to be relatively independent from the grid size, which is important for large computational meshes.

Multigrid Algorithm

- cycle\( (v, u_k, f, v_1, v_2) \)
- Smooth \( v \) times
- Compute residual \( r_k = f - L u_k \)
- Restrict residual \( r_{k-1} = R r_k \)
- Compute approximate solution \( v_{k-1} \):
  - "Direct Solve" if lowest level, otherwise
  - Repeat \( \gamma \) times: cycle\( (v, 0, r_{k-1}, v_1, v_2) \)
  - Pro-longate correction: \( u_k = u_k + \mathcal{P} v_{k-1} \)
- Smooth \( v_2 \) times

\( \gamma = 1 \) \hspace{1cm} \( \gamma = 2 \)

V-cycles \hspace{1cm} W-cycles

Multigrid Implementation

Four kernels: Laplacian, Restriction (full-weighting), Smooth (Jacobi or Red-Black Gauss-Seidel), and Prolongation.

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Question: Which of these parameters are most important to tune? Truncation level, cycle type, smoother type, method for coarsest grid solve.

Conclusions:
- Multigrid on GPUs can be effective and efficient.
- Multigrid performance significantly benefits from deeper truncation levels.
- The Fermi architecture makes a difference in relative smoother performance.
- Coarsest level operations have a large impact on cluster scalability.

Acknowledgments: NASA Idaho Space Grant Consortium, NVIDIA Professor Partnership Program, NCSA grant #ATM100032