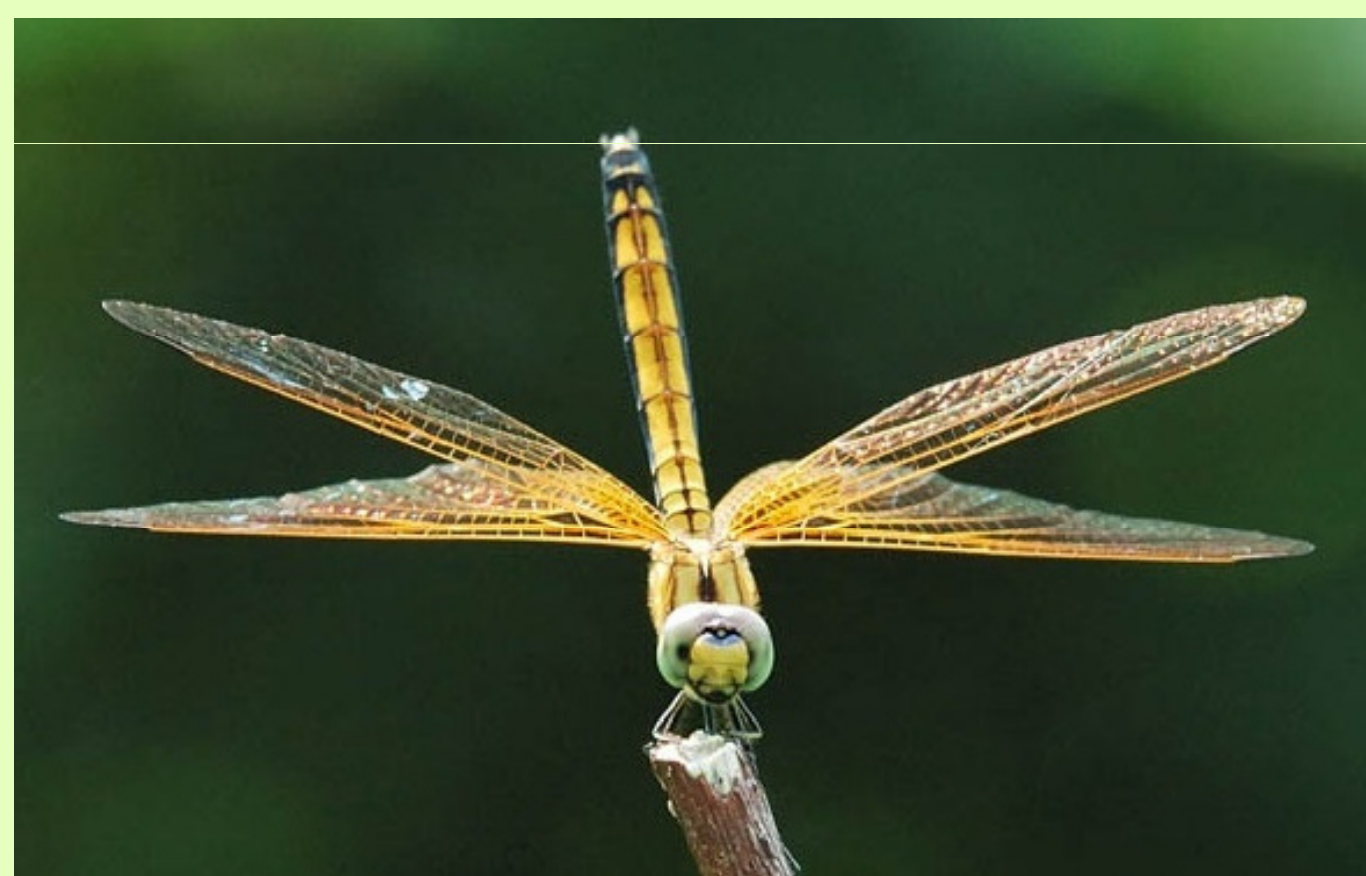
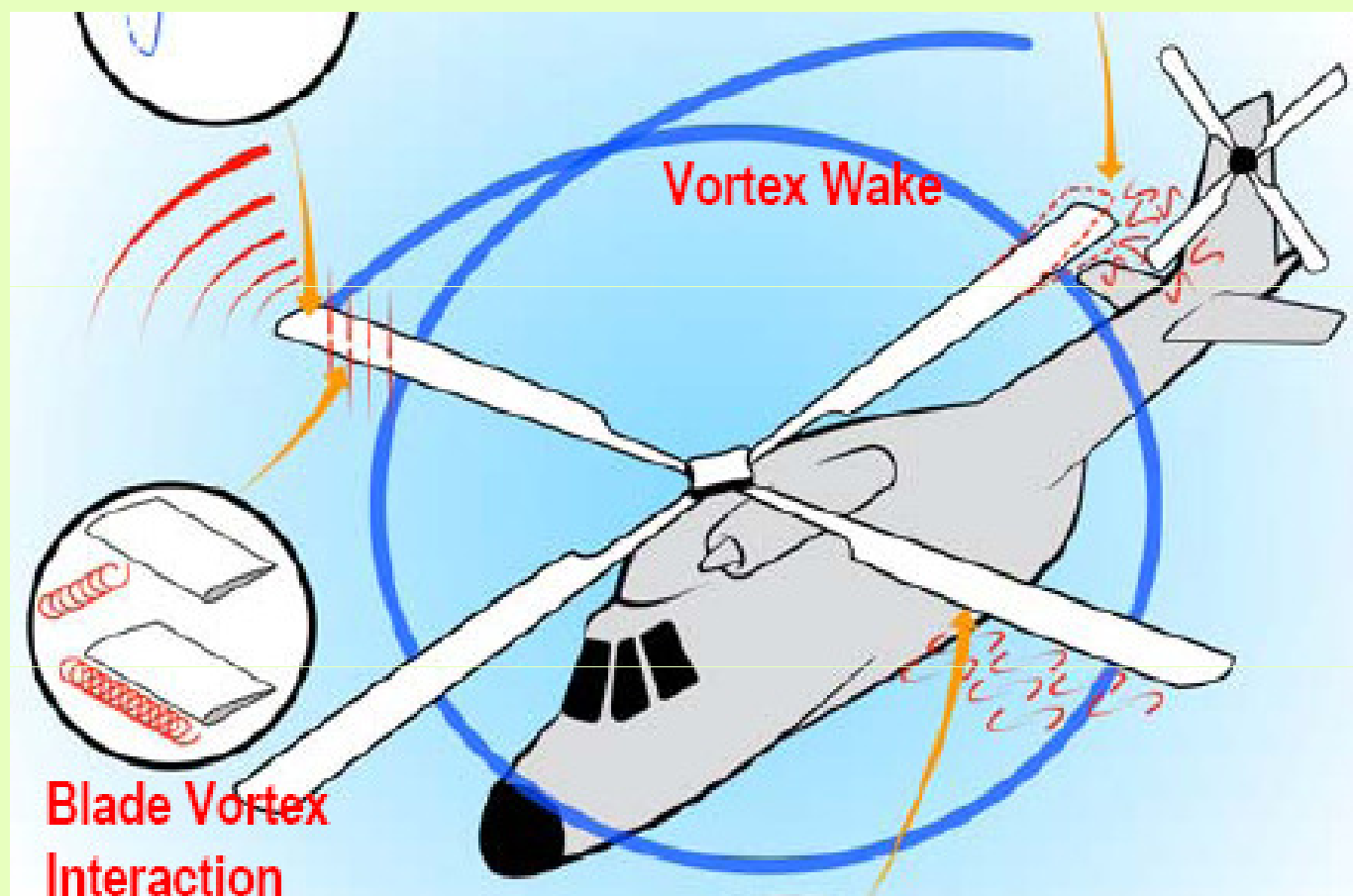


Overview

- ❖ Most production/commercial computational fluid dynamics codes are only 1st or 2nd order accurate
- ❖ Though adequate for a wide range of applications, many problems require higher-order accuracy. For example:
 - Aeroacoustic problems
 - Vortex dominated flow such as flow over a helicopter and a dragonfly
 - Direct numerical simulation and large eddy simulation of turbulent flow



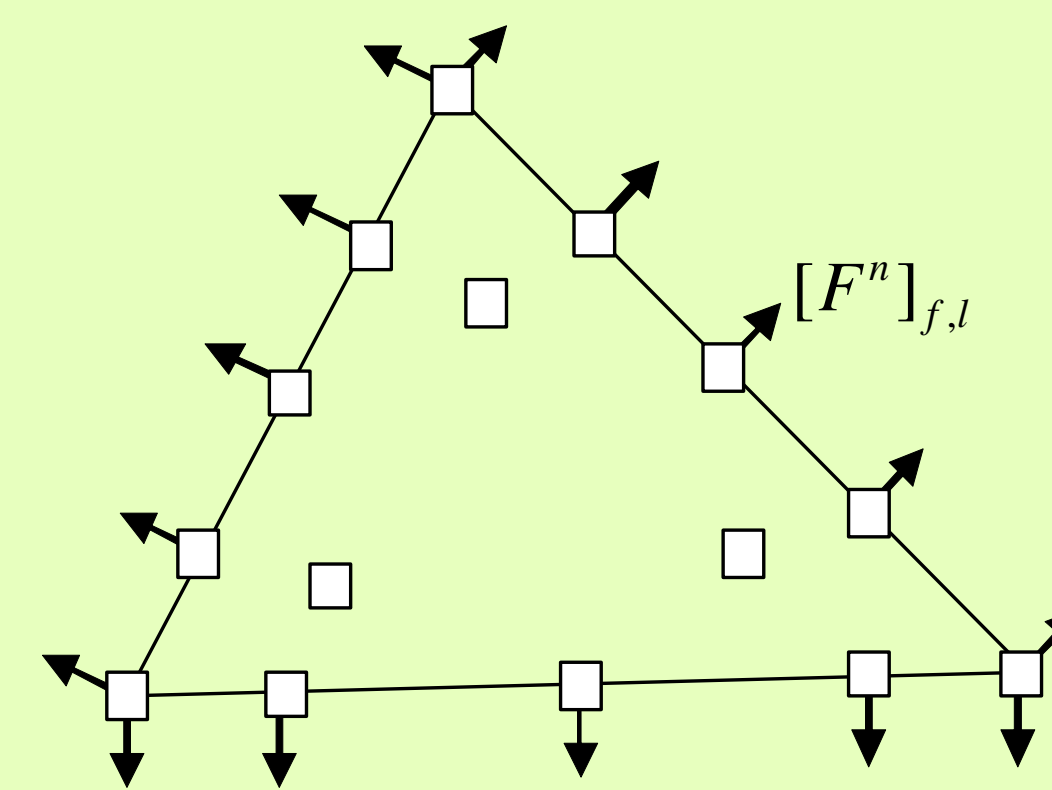
- ❖ Higher-order methods are notoriously expensive comparing with lower order ones. More efficient solvers sought after.
- ❖ GPGPUs provide a very promising platform for high-order methods as they are computationally intensive and compact.

High-Order CPR Formulation

Consider a hyperbolic conservation law

$$\frac{\partial Q}{\partial t} + \nabla \cdot \vec{F}(Q) = 0$$

The approximate solution at cell V_i is denoted Q_i , which is a degree k polynomial without continuity requirement across cell interfaces. The degrees of freedom (DOFs), $Q_{i,j}$, are the approximate solutions at a set of points named solution points, as shown in the next figure.



Solution Points, $k = 4$

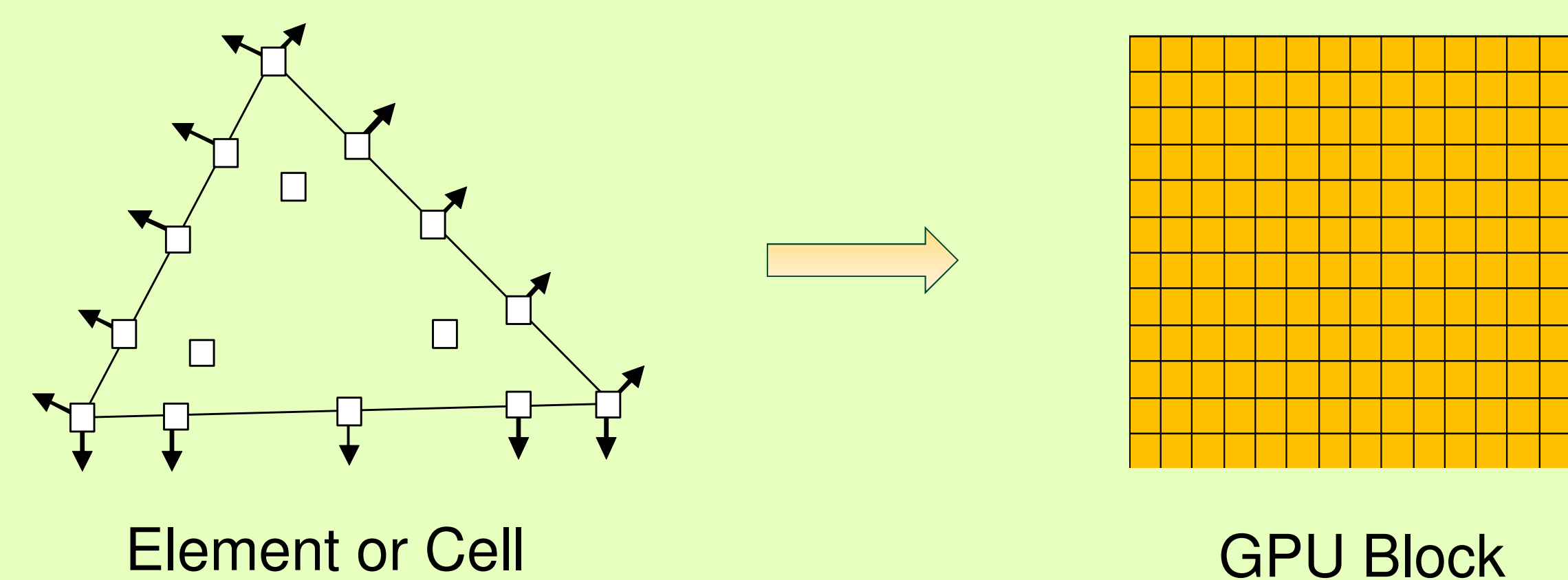
The DOFs are updated using a Correction Procedure via Reconstruction (CPR) formulation

$$\frac{\partial Q_{i,j}}{\partial t} = -\nabla \cdot \vec{F}(Q_{i,j}) - \frac{1}{|V_i|} \sum_{f \in \partial V_i} \sum_l \alpha_{j,f,l} [F^n]_{f,l} S_f$$

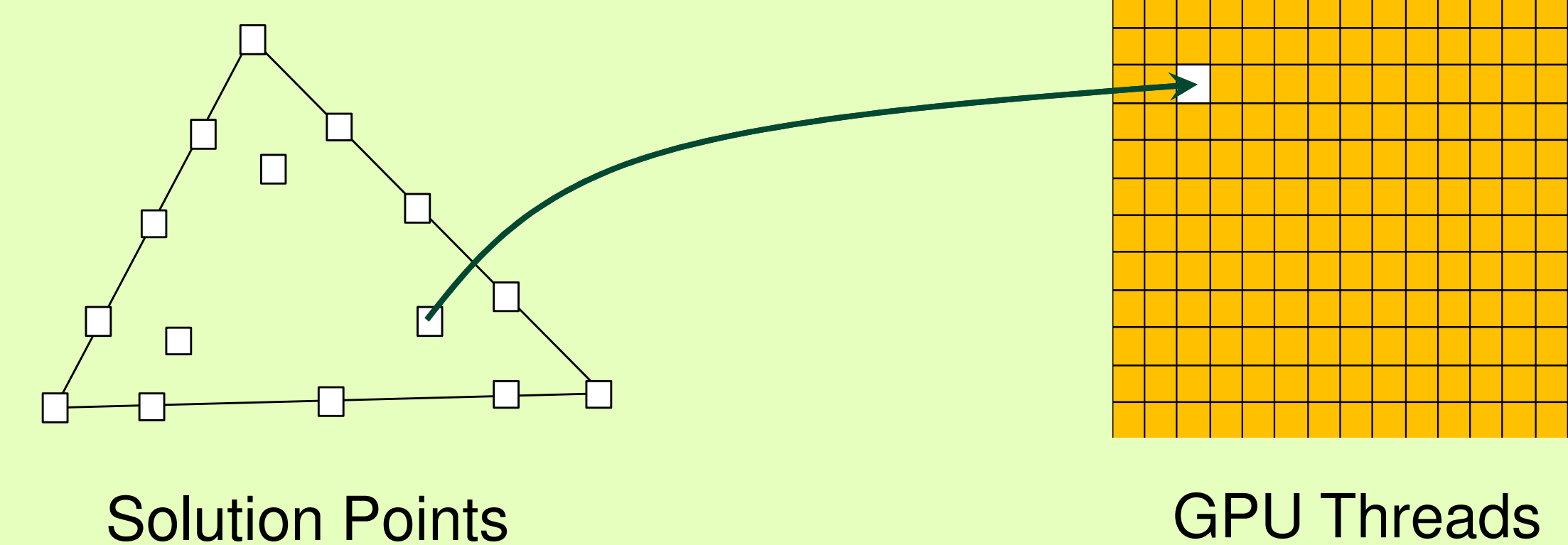
The first term on the RHS involves data only on the present element, and is therefore completely local. The second term (the correction term) needs data at neighboring elements sharing a face. This term provides coupling between neighboring elements and ensures stability.

Implementation on GPGPU

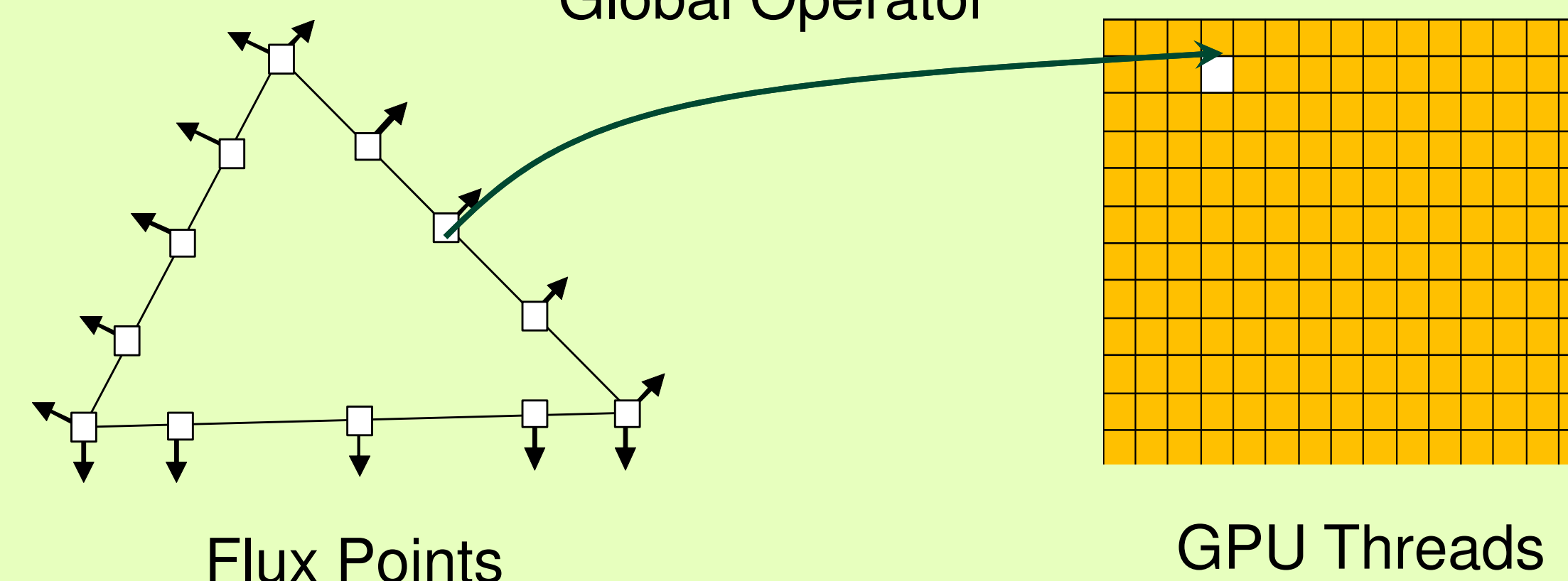
Data mapping



Local Operator

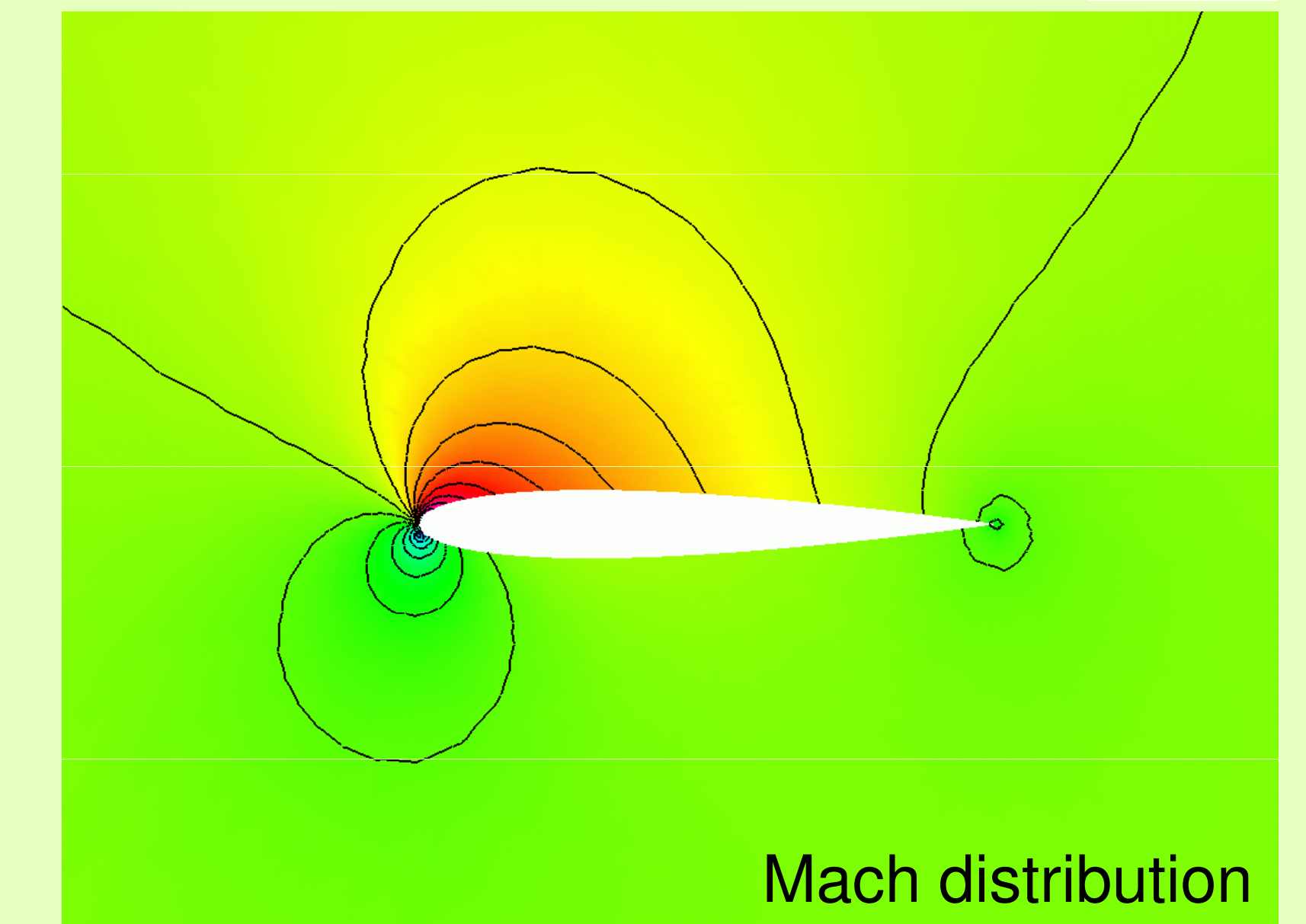
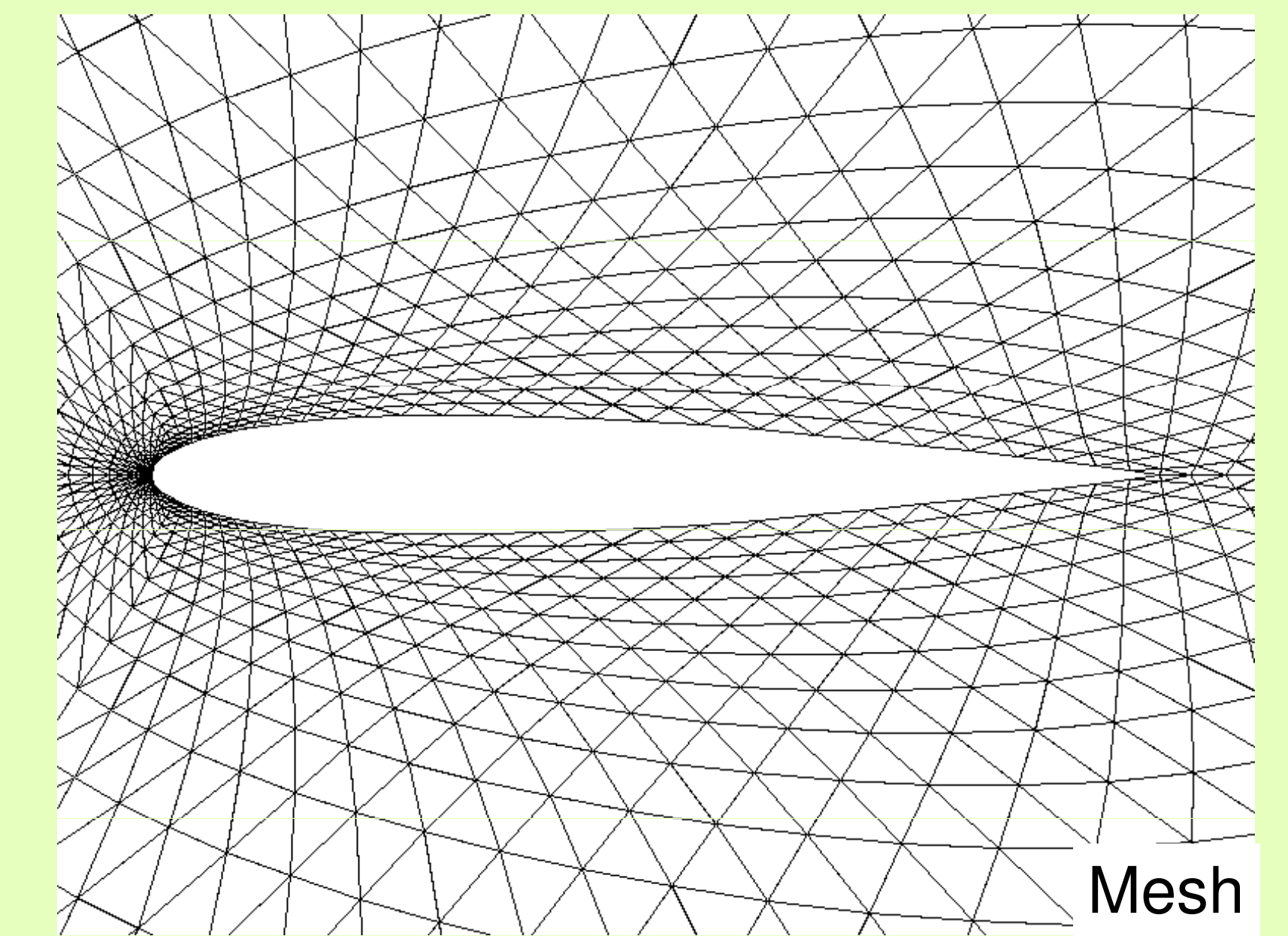


Global Operator



Test Case

Subsonic Flow over a NACA0012 Airfoil, $M = 0.3$, $\alpha = 1$ deg.



Speedup (CPU/GPU Time)



Conclusions and Future Work

- ❖ GPCPU is very effective for adaptive high-order methods. More than an order of magnitude of speedup obtained.
- ❖ Implementation for 3D viscous flows will be carried out next.

Acknowledgements

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